

Feature work

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We will study about region crossing change, region freeze crossing change and Reidemeister move on spherical curve.

For knots and links, there are many researches with invariants. In particular, Vassiliev[Va] introduced an invariant called Vassiliev invariant in 1990. It gives a certain classification of knot invariants. In fact, there is a conjecture called Vassiliev Conjecture;

For a pair of knots K_1 and K_2 , if K_1 does not equal to K_2 , then Vassiliev inv. distinguishes K_1 and K_2 .

K_1 equals to K_2 if and only if a diagram of K_1 obtained from a diagram of K_2 by Reidemeister move (see Figure 1). Then, on this move and Vassiliev invariant, we have following fact;

If a value of Vassiliev invariant for a diagram D_1 equals to a value of Vassiliev invariant for a diagram D_2 applied $\Omega 1$ and $\Omega 3$, then the values of Vassiliev invariant are the same.

Since the previous fact, we expected that if for a pair of link diagrams D_1 and D_2 , D_2 is obtained from D_1 by $\Omega 1$ and $\Omega 3$, then D_2 is obtained from D_1 by $\Omega 2$. However, Hagge-Yazinski[HY] gave an example from which the simple closed curve is NOT obtained by $\Omega 1$ and $\Omega 3$.

Viro [V] proposed Reidemeister moves of spherical curves called RI, weak RII, strong RII, weak RIII and strong RIII. Then we consider following problem;

We consider equivalence relations induced by a subset of {RI, weak RII, strong RII, weak RIII and strong RIII}. Then characterize equivalence classes by equivalence relations

For a projection G arising from a link diagram D , we define a 1-dimensional simplicial complex $C(G)$ as follows.

A 0-complex of $C(G)$ corresponds to an element of the power set of vertices of G . There is an edge e such that e connects two vertices v, v' of $C(G)$ if and only if there is a region R such that “the symmetric difference of the set of vertices of G corresponding to v , and the set of vertices of G corresponding to v' ” is changed by the region crossing change at R .

We develop the theory from the viewpoint “the shape of the simplicial complex”. In particular, we give a global understanding of studies about an efficiency of the switching system introduced by Akio Kawauchi-Kengo Kishimoto-Ayaka Shimizu, and Ayumu Inoue-Ryo Shimizu’s results [IS]. Particularly, Kawauchi-Kishimoto-Shimizu introduced a game based on region crossing change. We would link to introduce the difficulty and the efficiency of the switching system. Here is an example of the usage the complex: by a result of A. Shimizu, we see that $C(G)$ is connected (in example of the figure, we remark $C(G)$ contains 2 components). On the other hand, we define a simplicial complex $C'(G)$ by region freeze crossing change introduced by Inoue-Shimizu. Then results of Inoue-Shimizu is regarded as “there exists a projection represented of knot such that $C'(G)$ is not connected”. For a mutant of region crossing change, we obtain a simplicial complex from the operation. Then it shows some properties of the operation. Then we develop this philosophy.

Reference

- [Va] V. A. Vassiliev, Cohomology of knot space, in Theory of Singularities and its Applications (ed. V. I. Arnold), Adv. Soviet Math. 1, Amer. Math. Soc., 1990.
 [Vi] O. Viro, Generic immersions of the circle to surfaces and the complex topology of real algebraic curves. Topology of real algebraic varieties and related topics, 231–252, Amer. Math. Soc. Transl. Ser. 2, 173, Adv. Math. Sci., 29, Amer. Math. Soc., Providence, RI, 1996.

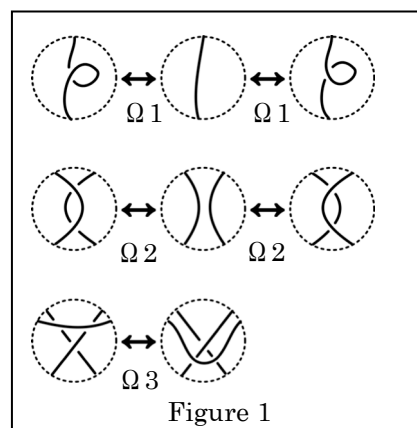


Figure 1

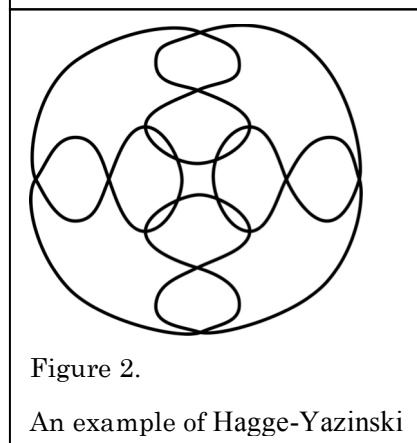


Figure 2.

An example of Hagge-Yazinski