Research Plan

(1) An explicit presentation of the cohomology rings of regular nilpotent Hessenberg varieties

We have already obtained an explicit presentation of the cohomology rings of regular nilpotent Hessenberg varieties in classical Lie types and type G_2. We first try to obtain an explicit presentation of the cohomology rings of regular nilpotent Hessenberg varieties in types E_6, E_7, E_8, F_4. Our goal is to give an explicit presentation of the cohomology rings of regular nilpotent Hessenberg varieties in all Lie types in terms of Lie theory.

(2) A basis of the cohomology of a regular nilpotent Hessenberg variety

We have already obtained a basis of the cohomology of a regular nilpotent Hessenberg variety in Lie types A, B, C, G_2 in terms of positive roots. We first try to obtain a basis of the cohomology of a regular nilpotent Hessenberg variety in types D, E_6, E_7, E_8, F_4 in terms of positive roots. Our goal is to give a basis of the cohomology of a regular nilpotent Hessenberg variety in all Lie types in terms of positive roots.

(3) Harada-Tymoczko conjecture

It is well-known that Schubert classes form a basis for the cohomology of a flag variety. Harada and Tymoczko conjectured that a basis for the cohomology of a regular nilpotent Hessenberg variety can be obtained by taking the Schubert classes σ_w for permutations w such that w belongs to the Hessenberg space. Our goal is to solve Harada-Tymoczko conjecture.

(4) Schubert calculus on a regular nilpotent Hessenberg variety

We try to do Schubert calculus on a regular nilpotent Hessenberg variety. More specifically, our goal is to introduce a Hessenberg version of Schubert polynomials and to calculate the intersection number of the closure of the Schubert cells in a regular nilpotent Hessenberg variety.