Research Plan

María de los Angeles Guevara Hernández

In order to classify and study links, these can be divided into alternating and non-alternating depending if they possess an alternating diagram or not, respectively. For alternating links, the invariants generally are easy to compute and actually, their forms are better understood. However, non-alternating links have geometric and topological structures more complex and interesting, which are not completely understood. In order to have a deep compression about non-alternating links and to extend some properties known for alternating link to more general classes, many generalizations have been given. Further, after the proof of the Tait flype conjecture on alternating links given by Menasco and Thistlethwaite, it became an important question to ask how a non-alternating link is “close to” alternating links under a metric. The Gordian distance between two links is the minimal number of crossing changes needed to deform one into the other; however, several different Gordian distances have been defined by using others local moves, for instance a band surgery or SH(3)-move are local moves in a link diagram. Furthermore, since some enzymes are responsible of local changes in DNA, Gordian distances can be used to study their actions.

By considering crossing changes, Adams et al. in 1992 introduced the dealternating number of a link L as the minimum number of crossing changes necessary to transform a diagram D of L into an alternating diagram. Besides, Kawauchi in 2010 introduced the alternation number of a link L as the minimum number of crossing changes necessary to transform a diagram D of L into some (possibly non-alternating) diagram of an alternating link. Both link invariants are zero if and only if the link is alternating, further it is immediate from their definitions that $alt(L) <= dalt(L)$ for any link L. In general, it is hard to determine these invariants for knots and the difference between dealternating number and alternation number lies in whether the Reidemeister moves are admitted after a crossing change or not.

Recently Lowrance demonstrated that there exist families of links for which the difference between certain alternating distances is arbitrarily large. In particular, he proved for each positive integer n there exists a satellite knot K such that $alt(K) = 1$ and $dalt(K)<=n$. In addition to previous results, we gave an infinite family of hyperbolic prime knots such that $alt(K) = 1$ and $dalt(K) = n$, instead of just one knot. Moreover, all the knots in each family have $dalt(K) = n$.

The purpose of proposed research is to enhance the classification of non-alternating knots, and to improve the tabulation of Gordian distances between two knots. In order to accomplish this goal, since that $alt\ (L)$ and $dalt(L)$ are not sufficiently studied, we would like achieve the following:

1.-Deepen the study of the alternation number and others related invariants. Abe gave a lower bound for the alternation number of a knot by using the Rasmussen invariant (which is derived from the Khovanov homology). Actually, there is not another type of lower bound, but we would like to show that this inequality is not sharp for some knots.

2.-Improve the tabulation of the alternation number and the dealternating number of prime knots. These invariants have been calculated for some knots; however, for all prime knots up to twelve crossings these invariants have not been completely determined.

3.- Advance in the tabulation of the minimum number of crossing changes between any two knots. Kanenobu has obtained results to improve the knots table of up to seven crossings. However, there are still undetermined values.