(2) Abstract of results Makiko Mase

(a) **On birational equivalence among families of $K3$ surfaces**

Among many $K3$ surfaces that exist, they appear as parametrised by the complete anti-canonical linear system of Fano weighted projective spaces, and of smooth Fano 3-folds.

In [Kobayashi-Mase, 2012], [Mase, 2012], and [Mase, 2014], it is concluded that some of the families of $K3$ surfaces with isometric Picard lattices are birationally equivalent. Here, instead of using Torelli-type theorem, an explicit monomial map between two general sections in families is constructed.

(b) **On duality of families of $K3$ surfaces and strange duality of singularities**

It is observed by Ebeling and Takahashi, and Ebeling and Ploog that there exists a strange duality for invertible polynomials. In [Mase-Ueda, 2015], and [Mase, 2016–17], is is shown that the strange duality for bimodal singularities can extend to a Batyrev-Borisov mirror symmetry for families of $K3$ surfaces. Morever, if the ambient space is simplicial, the mirror symmetry extends to a duality of Picard lattices. The isolated hypersurface singularities in $\mathbb{C}^3$ under dealt admit a projectivisation that is given by an invertible polynomial.

(c) **On Weierstrass semigroup in pointed curves in a $K3$ surface**

In a submitted paper [Curves on weighted $K3$ surfaces of degree two with symmetric Weierstrass semigroups, J.Komeda and M.Mase], it is concluded that in certain $K3$ surfaces, there exist curves that admit given Weierstrass semigroups.

(d) **Families of $K3$ surfaces and sextic curves of $(2,3)$-torus type**

It is studied by Tokunaga et. al. if we consider a double covering of the projective plane branching along a $(2,3)$-torus type, and in this case only, we obtain a cyclic triple covering from a Gorenstein $K3$ surface, which is the normalisation of the non-Galois triple covering of the projective plane to a $K3$ surface that is a double covering of $\mathbb{P}^2$.

Motivated by this, a submitted paper [Families of $K3$ surfaces and curves of $(2,3)$-torus type] focuses on families of $K3$ surfaces obtained as a double covering of the projective plane branching at a $(2,3)$-torus type.

Sextic curves of $(2,3)$-torus type are classified by Oka and Pho together with their defining polynomials. Thus we explicitely describe families of $K3$ surfaces as sublinear system of the complete anticanonical linear system of the weighted projectie space with weights $(1,1,1,3)$.

In the first part, we study dualities of polytopes and of Picard lattices for the families. And in the second part, we study families that contain $K3$ surfaces that admit each singularity from the branch curve.

Since the Picard lattices are generated by algebraic subvarieties of a $K3$ surface, it is expected that this result may also characterise them.