Results of my research

Shin’ya Okazaki

A genus $g$ handlebody-knot is a genus $g$ handlebody embedded in the 3-sphere, denoted by $H$. Two handlebody-knots are equivalent if one can be transformed into the other by an isotopy of $S^3$. Cutting along a meridian disk system of $H$, we have a knotted solid torus in $S^3$. A constituent knot of $H$ is the knot which is the spine of the knotted solid torus. The constituent knot depend on choice of a meridian disk. There are infinite many meridian disks for a handlebody-knot. Thus, there are infinite many constituent knots for a handlebody-knot. Let $CK(H)$ be the set of all of constituent knots of $H$.

Litherland introduced another version of the Alexander polynomial for $\theta_g$-curves [2]. Litherland’s Alexander polynomial of a $\theta_g$-curve includes information of the constituent knots of the $\theta_g$-curve. We extend Litherland’s Alexander polynomial of a $\theta_g$ to that a pair of $H$ and its meridian system with base point.

Let $K$ be a knot in $S^3$. The Nakanishi index $m(K)$ of $K$ is the minimum size among all square Alexander matrix of $K$. Let $\Delta_K(t)$ be the Alexander polynomial of $K$, that is, g.c.d. of the $(n-d+1)$-minor of an $m \times n$ presentation matrix of the first homology group of the universal abelian covering of the exterior of $K$. We have the following theorem.

**Theorem 1** [O.]

$K \in CK(4_1) \Rightarrow \ m(K) \leq 1$ or $\Delta_K(t)$ is reducible.

Here, $4_1$ is the handlebody-knot in the table of genus 2 handlebody-knots with up to six crossings in [1]. We have that the knot $9_{35}$ is not a constituent knot of the handlebody-knot $4_1$ by Theorem 1.

**References**
