Results of my research

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A genus g handlebody-knot is a genus g handlebody embedded in the 3-sphere, denoted by H. Two handlebody-knots are equivalent if one can be transformed into the other by an isotopy of S^3. Cutting along a meridian disk system of H, we have a knotted solid torus in S^3. A constituent knot of H is the knot which is the spine of the knotted solid torus. The constituent knot depend on choice of a meridian disk. There are infinite many meridian disks for a handlebody-knot. Thus, there are infinite many constituent knots for a handlebody-knot. Let $CK(H)$ be the set of all of constituent knots of H.

Litherland introduced another version of the Alexander polynomial for θ_g-curves [2]. Litherland's Alexander polynomial of a θ_g-curve includes information of the constituent knots of the θ_g-curve. We extend Litherland's Alexander polynomial of a θ_g to that a pair of H and its meridian system with base point.

Let K be a knot in S^3. The Nakanishi index $m(K)$ of K is the minimum size among all square Alexander matrix of K. Let $\Delta_K(t)$ be the Alexander polynomial of K, that is, g.c.d. of the $(n-d+1)$-minor of an $m \times n$ presentation matrix of the first homology group of the universal abelian covering of the exterior of K. We have the following theorem.

Theorem 1 [O.]
$K \in CK(4_1) \Rightarrow m(K) \leq 1$ or $\Delta_K(t)$ is reducible.

Here, 4_1 is the handlebody-knot in the table of genus 2 handlebody-knots with up to six crossings in [1]. We have that the knot 9_{35} is not a constituent knot of the handlebody-knot 4_1 by Theorem 1.

References

[1] A. Ishii, K. Kishimoto, H. Moriuchi, and M. Suzuki, *A table of genus two handlebody-knots up to six crossings*, Journal of Knot Theory Ramifications **21**, No. 4, (2012) 1250035, 9 pp.

[2] R. Litherland, *The Alexander module of a knotted theta-curve*, Math. Proc. Camb. Phil. Soc. **106** (1989), 95–106.