The research so far

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An approach to the theory of harmonic maps via exponentially harmonic maps

The applicant has *reestablished some existence theory of harmonic maps by an unified approach using exponentially harmonic maps.*

Harmonic maps are defined as critical points of the energy functional $\mathrm{E}(u) = \int_M |\nabla u|^2 \, d\mu_g$ for maps $u\colon M \to N$ between two Riemannian manifolds. The functional is a generalization of the Dirichlet integral, which defines harmonic functions. Harmonic maps are one of the most fundamental research objects in the area of geometric analysis, and their existence is known to depend deeply on the dimension of the source manifold, the geometry or the topology of the target manifold, etc. On the otheer hand, for $\varepsilon > 0$, an ε-exponentially harmonic map is defined as a critical point of

$$\mathbb{E}(u) = \int_M e^{\varepsilon |\nabla u|^2} \, d\mu_g,$$

and, independently of the geometry of manifolds, its full existence and regularity are known in various situations. The applicant has established the following. (The numbers correspond to those in the papers list.)

- In the case that N is nonpositively curved, a sequence (u_ε) of ε-exponentially harmonic maps is shown to converge uniformly to a harmonic map as $\varepsilon \to 0$, which reproves Eells-Sampson's existence theorem for harmonic maps [1].
- In the case that $\dim M = 2$, the obstruction, called bubbles, to the convergence of (u_ε) is studied, and it is proved that the sequence converges uniformly to a harmonic map provided $\pi_2(N) = 0$, which reproves Sacks-Uhlenbeck's existence theorem [2].
- He has also proved an existence theorem for exponentially harmonic maps in the case that M is noncompact, and obtained a Liouville-type theorem for exponentially harmonic maps with bounded energy whose target has nonpositive curvature [4].
- The existence of a time-global solution to a time evolutional equation for exponentially harmonic maps into nonpositively curved manifolds has been proved [5].
- The existence of a kind of equivariant exponentially harmonic maps between spheres has been proved under without any conditions such as damping conditions, which are necessarily needed for the existence of equivariant harmonic maps. This study is a joint work with Y.-J. Chiang and H. Urakawa [7].

A discrete surface theory for graphs

The applicant also studies, *from a viewpoint of the area of material science, some realizations of graphs in $\mathbb{R}^3$, a discrete surface theory for them, and a continuous limit of their subdivisions.*

- In the jointwork [3] with M. Kotani and H. Naito, the applicant has proposed a discrete surface theory on 3-valent embedded graphs in $\mathbb{R}^3$ which are not necessarily “discretization” or do not have “faces”. The definitions of the normal vector, the mean and the Gauss curvature are proposed from a viewpoint of the discrete geometric analysis. This study is new in that it deals also with non-polyhedral surfaces.
- In the jointwork [6] with H. Naito and T. Tate, he also studied spectral problems of the Goldberg-Coxeter subdivisions (GC subdivisions) for 3- and 4-valent finite graphs. The GC subdivision has parameter k and the number of its vertices is increasing in order $O(k^2)$ nevertheless its girth is bounded. In [6], they have detected several universal eigenvalues for the Laplacian of the GC subdivision, and the first and the last $o(k^2)$ eigenvalues are shown to converge to 0 and the natural upper bound 6 or 8.