今後の研究計画の英訳

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(1) Pseudo-reflection groups

Nontrivial automorphisms on a complex vector space which fix a hyperplane pointwisely is called pseudo-reflections. In the real case, the degree of such automorphism is always 2 and called a reflection simply, but those of pseudo-reflections may be more than 2. A finite group is called a pseudo-reflection group when it is generated by pseudo-reflections. Pseudo-reflection groups are natural generalization of Weyl groups, so their combinatorics and geometry must relate each other. So I hope to reveal this relation.

The classification of pseudo-reflection groups is completed by Shephard and Todd more than 60 years ago, however its combinatorics is almost not known. Keywords to understand Weyl groups are a length function and a poset structure, however they are not known for pseudo-reflection groups. Under some completion, there are analogues of flag varieties for pseudo-reflection groups, however combinatorial structure such as Bruhat decomposition is not known. My knowledge on the relation among the Weyl group, the cohomology ring of the flag variety, and the Bruhat decomposition can help me to find these combinatorial structures. Concretely, I will try to construct geometrical object as something like a cell complex by calculating the attaching map from the GKM-theoretical description of the "equivariant cohomology ring" corresponding a pseudo-reflection group.

There are some obstacle to overcome. Firstly, the known GKM-theoretical description is not constructive, and I have to rewrite it in a form which indicates a structure of a cell decomposition. Secondly, I have to define a length function and a partial order. Now I am considering to rephrase the length function and the Bruhat order of a Weyl group to apply them to pseudo-reflection groups. And I confirmed that my rephrasing works well for some small pseudo-reflection groups in the sense that the length distribution coincides with that of Betti numbers of the "cohomology ring". There are some pseudo-reflection groups which are called Shephard groups. A Shephard group is written as a group of symmetries of some figure in a complex vector space, so it may be possible to give direct definitions based on the combinatorics of such figures.

(2)Regular semisimple Hessenberg varieties

A regular semisimple Hessenberg variety is closed under the maximal torus action, and the tangent space at some T-fixed point is the subspace corresponding to the lower ideal. A regular semisimple Hessenberg variety has a paving rather than a cell decomposition, and its equivariant cohomology ring also has a GKM-theoretical description, but it is difficult to obtain an explicit description as a quotient ring of a polynomial ring. The cohomology classes corresponding to each cells is not known. I want to give an explicit description of the cohomology ring of a regular semisimple Hessenberg variety in terms of a lower ideal and an description of the cohomology classes corresponding to each cells.

For a fixed lower ideal, one obtain a fiber space whose base space is the Lie algebra of G and whose fiber is the Hessenberg variety. I want to combine my results and a description of the cohomology rings of regular semisimple Hessenberg varieties.