これまでの研究成果の英訳

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(1) Motivation of my research

Flag maniflods have high symmetry and their geometric property is often described in terms of combinatorics. For example, the rational cohomology ring of a flag manifold is the coinvariant ring of the corresponding Weyl group, the poset structure of the Weyl group determines the Bruhat decomposition of the flag manifold, and the complex dimension of the cell is equal to the length of the corresponding element of the Weyl group. The aim of my research is to reveal the relation of combinatorics and other geometrical objects.

(2) Results of my research

Let G be a compact connected Lie group and T be a maximal torus of G. The maximal torus T acts on the flag manifold G/T by the left multiplication. For G is F_4 or E_6, I determined the T-equivariant cohomology ring with integer coefficients of the flag manifold G/T as the quotient ring of the polynomial ring by the ideal generated by explicit elements.

I also determined the T-equivariant cohomology ring with integer coefficients of the flag manifold of type C. I employed the GKM theory to determine these cohomology rings. The GKM theory for flag varieties is understood well in the point of view of the Bruhat decomposition, and it describes the attaching maps in terms of the root system. My calculation was based on the combinatorial structure, so I gave geometrical meaning to my representation of the cohomology ring.

Hessenberg varieties are subvarieties of the flag variety G/T. Let Φ^+ be the positive root system of G. A Hessenberg variety is determined by two data; an element of the Lie algebra of G and a “good” subset of Φ^+ （this is called a lower ideal）. When $I = \Phi^+$, the Hessenberg variety is the flag variety. It was known that Hessenberg varieties includes Springer varieties, so Hessenberg varieties are important. I determined, with other researchers, that the cohomology ring of a regular nilpotent Hessenberg variety as a quotient ring of a polynomial ring in terms of a lower ideal. Moreover we proved that the ring of the Weyl group invariant elements of the cohomology ring of a regular semisimple Hessenberg variety is isomorphic to the cohomology ring of the regular nilpotent Hessenberg variety.

Above results indicate that the combinatorics of the lower ideal appears as the geometry of the regular nilpotent Hessenberg variety. I will explain this phenomenon to emphasize the importance of Hessenberg varieties. Cells of the Bruhat decomposition of a flag variety correspond to elements of the Weyl group, and the complex dimension of a cell is equal to the length of the corresponding element. The Lie algebra of T has a hyperplane arrangement defined by the positive root systemΦ^+, a connected component, that is, a Weyl chamber corresponds to an element of the Weyl group, and the combinatorics of the hyperplane arrangement describe the poset structure of the Weyl group. Since a lower ideal is a subset of Φ^+, one obtain subarrangement and its combinatorics give a cell decomposition of a regular nilpotent Hessenberg variety in the same manner.

In this point of view, Hessenberg varieties are important and beautiful subvarieties. Therefore our result is important, which determined their cohomology rings and reveal the relation of combinatorics and geometry of regular nilpotent Hessenberg varieties.