(2) Research achivements

My current research interest is in analysing the *Killing equation* from the perspective of general relativity and of the problem of determining whether a given Hamiltonian system is completely integrable or not. In my previous research, I undertook and partially answered the following questions:

- Are there any solutions of the Killing equation for given metrics?
- If the answer is yes, then how many solutions are there?
- What quantities are sufficient to determine the number of solutions?

Main results were as follows:

(i) We have constructed a procedure which transforms the Killing equation of a specified order into a closed system dubbed the *prolonged system* by introducing new variables. Then the explicit form of the prolonged system was written out up to third-order [4]. I did the same up to fourth-order in my PhD thesis. A key ingredient was a projection operator called *Young symmetriser*.

(ii) We have derived the concrete form of the integrability conditions of the prolonged system that impose strong restrictions on the Riemann curvature tensor and its derivatives. The concrete form of the integrability conditions are succinctly expressed in the Young symmetrisers [4].

(iii) We have characterised metrics which admit Killing vector fields by *local curvature obstructions*. The obstructions have been obtained by analysing the integrability condition and we saw that they can be expressed as the *curve(surface)-theoretic parameters*. For instance, the geodesic, normal curvature and the relative torsion of the eigensystem of the Ricci tensor are the obstructions for some cases. We have provided the branching algorithm that tells us how many Killing vector fields exist for a given 3-dimensional metric in my PhD thesis.

(iv) We have generalised Killing tensor fields in order to describe first integrals that are rational in momenta [3]. We introduce the notion of inconstructible generalised Killing tensors, which cannot be constructed from ordinary Killing tensor fields. Moreover, we introduce inconstructible rational first integrals, which are constructed from inconstructible generalised Killing tensor fields, and provide a method for checking the inconstructibility of a rational first integral.