Research Plan

Properties of blow-up solutions to Neri type mean field equation

I plan to study mean field equation derived by C. Neri. Sawada-Suzuki type is a turbulence model which is controlled by a deterministic distribution of vortex intensity and Neri type is one which is controlled by a stochastic distribution of it. Since Neri type equation also has a variational structure and scale invariant, it is natural to study it by using blow-up analysis. However, there are no works to do so. In my plan, I will formulate a process of blow-up analysis for Neri type equation and study a property of blow-up solutions on applications. In particular I shall study asymptotic nondegeneracy of blow-up solutions to Neri type equation. Asymptotic nondegeneracy of blow-up solutions means that the linearlized problem of Neri type equation around blow-up solution admits only trivial solution. In the following I explain the detail of my plan.

Aims of this work

(a) Asymptotic behavior of blow-up solutions to Neri type equation
(b) Asymptotic nondegeneracy of blow-up solutions to Neri type equation

(a) I will derive the asymptotic behavior of blow-up solutions to Neri type equation. For this, I employ the scaling argument and obtain a mass identity which is a relation about scaling limit. To obtain such a identity, I need to study the classification of entire solution to a scaling limit equation. The argument of scaling is discussed in Marchis-Ricciardi' 17 and it is reduced to entire solutions for Chen-Li type equation. Furthermore, total mass of scaling limit is 8π. This coincides with the value of mass of delta measure concentrating at blow-up point which is the limit of nonlinearity of Neri equation in a sense of measure. From these facts, I expect to derive a mass identity precisely. If mass identity holds, I can derive the asymptotic behavior by the argument in my previous work. Furthermore, I expect to obtain shaper result of Neri equation than Sawada-Suzuki one. To do so, I need to investigate the accuracy of convergence of mass after scaling. I refer to the original paper of C. S. Lin which is concerned with the accuracy of convergence of limiting mass for exponential elliptic equation and clear the argument in this work.

(b) I will show the asymptotic nondegeneracy of blow-up solutions to Neri type equation on applications of the asymptotic behavior of it.

The study of asymptotic nondegeneracy of blow-up solutions to elliptic equation is known in Gladial-Grossi' 04. This work deals with Gel'fand problem which is a elliptic equation with exponential nonlinearity. The proof is based on leading a contradiction by assuming to exists a nontrivial normalized solution for linearized eigenvalue problem. After scaling this nontrivial solution to the linearized problem, the entire solution is expressed explicitly. In fact such a solution is written by the linear combination of some three functions, see Baraket-Pacard' 98. As the result, by applying the the asymptotic behavior of family of blow-up solutions, the coefficients of the linear combination are zero and it leads a contradiction. I will consider this argument to apply the problem of Neri type equation. Recently, the asymptotic nondegeneracy of blow-up solutions to mean field equation is studied in Bartolucci-Jevnikar-Lee-Yang' 18. The argument in this work will also be valid for the equation with probability measure.