My Research Results

I have studied the property of solutions to several nonlinear elliptic equations by using blow-up analysis. In particular, I dealt with the following two problems.

1. A priori bounds for positive solutions to nonlinear elliptic equations with inhomogeneous function

I studied a property of positive solutions to some semilinear elliptic equation with inhomogeneous coefficient function. This equation describes the equilibrium state of nonlinear phenomena such as heat diffusion and motion of cellular slime mold so that many works have been done. Main problem for this equation is the existence of solutions and it is reduced to whether a priori bounds for solutions hold or not. In the previous work of Gidas-Spruck, a priori bounds was studied for the case the coefficient function is uniformly positive, but it was not clear for the case its zero points exist. Thus I studied a priori bounds for positive solutions to this equation in case the coefficient function has zero points.

The difficulty is caused by the behavior of the coefficient function near its zero points. This is because if it is strictly positive then a priori bounds is obtained by using some structure of the equation, however such a method is no useful in case the coefficient function has zero points due to its behavior around zero points. By applying the scaling invariants of the equation, I introduced the transformation depending on the behavior around zero points and derived a priori bounds for positive solutions.

In addition to a priori bounds, we obtained a new Liouville-type theorem in this work. This theorem says a property of entire solutions to some elliptic equation and plays an essential role to derive a priori bounds. In fact, by using the new transformation described in above, the original equation is reduced to the problem in a whole space and we can apply Liouville-type theorem for it. This theorem was proved by the moving plane method which is valid for deriving the symmetry of solutions with respect to the space.

2. The behavior of blow-up solutions to mean field equation with probability measure

I studied a property of solutions to mean field equation with probability measure. Mean field equation is a two dimensional semilinear elliptic equation which has the exponential nonlinearity and plays a mathematical modeling about a turbulence phenomenon such as great red spot on surface of the Jupiter. In particular, I dealt with mean field equation derived by Sawada-Suzuki. The difference of the model of Sawada-Suzuki and another model (Caglioti-Lions-Marchioro-Pulvirenti' 91) appears in the distribution of vortex intensity assumed in derivation.

In general, mean field equation has a variational structure and it is studied by many mathematician. Thus there exists a energy functional corresponding to mean field equation and its critical point means the weak solutions to the equation. Concerning a critical point, properties of function attains the minimum of it are important. In my work, I focus on such a minimizer of the energy functional corresponding to Sawada-Suzuki type mean field equation, in particular studied the asymptotic behavior of unbounded family of minimizers. Such a situation is called concentration and it means the nonexistence of solution in a sense of critical case. By combination of blow-up analysis and potential theory, I succeeded to derive the asymptotic behavior of unbounded sequence of minimizers.