Research Proposal

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1. Research on local times via the Tanaka formula

We have already obtained the Tanaka formula for stable processes and the formula for Lévy processes. Then, by using the formula, we study the local time for Lévy processes and investigate its properties and structure.

The necessary and sufficient conditions for the existence of local times for Lévy processes, and the necessary and sufficient conditions for the joint continuity of local times are known via a potential theoretic approach, but we investigate the continuity of local times via the Tanaka formula. By using Itô's stochastic calculus, we have obtained the explicit formula for stable processes, but we could not express the martingale part in Tanaka formula for Lévy processes via the potential theoretic approach. Thus, we will construct the formula for Lévy processes via Itô's stochastic calculus. From the formula, we investigate the relation between the Lévy processes and its local time. On the other hand, by using the potential theoretic techniques, we will construct the formula for Markov processes.

2. On pathwise uniqueness for stochastic differential equations

We will study the pathwise uniqueness of the solutions for the stochastic differential equations driven by one-dimensional Lévy processes under the condition on the coefficients. We investigate the Hölder conditions on the diffusion and drift coefficients under which the pathwise uniqueness holds. On the basis of the techniques in Fournier (2013), we obtained the Hölder conditions on coefficients under which the pathwise uniqueness holds when the driving process is in more general class. Thus, we investigate the conditon on coefficients if the driving process is a stable process with index $\alpha \in (0, 1]$, a stable-like process, or multi-dimensional Lévy processe and we study the rate of convergence of the Euler–Maruyama scheme. Furthermore, by using the techniques based on local times in Komatsu (1982) and Le Gall (1983) and our Tanaka formula, we study the pathwise uniqueness.