Research Results

Name: Hiroshi TSUKADA

We study the local time for Lévy processes and obtain the Tanaka formula for stale processes via Itô's stochastic calculus and the formula for Lévy processes via the potential theoretic approach, and study the solution of stochastic differential equations driven by Lévy processes and obtain the Hölder condition on coefficients under which the pathwise uniqueness of solutions of the stochastic differential equations holds.

■ Local time and Tanaka formula for Lévy processes

The local time characterizes the amount of time spent by the process at a given level. For Brownian motions, the representation which is called the Tanaka formula is well known. This formula represents the Doob–Meyer decomposition for local times, that is, the local time can be understood as the increasing process given by the difference of the submartingale and the martingale. Thus, we constructed the Tanaka formula for Lévy processes from viewpoint of the Doob–Meyer decomposition.

For stable processes, we focused on an occupation time formula and constructed the Tanaka formula via Itô's stochastic calculus. By using the Fourier analysis, we obtained the fundamental solution of the infinitesimal generator for stable processes, and applied the Itô formula, and then we obtained the Tanaka formula for stable processes. We obtained that the martingale part in Tanaka formula is square integrable from the moment estimates for stable processes.

It was difficult to find the fundamental solution of the infinitesimal generator for Lévy processes. Thus, from the connection between the local time and the resolvent density in Blumenthal–Getoor (1964), we constructed Tanaka formula for Lévy processes. The renormalized zero resolvent was an important point to construct the formula. In Yano (2013), they needed some conditions for the existence of the renormalized zero resolvent for Lévy processes, but it was difficult to check the conditions. Our result also gave the existence of the renormalized zero resolvent under weaker conditions then the ones in Yano (2013). It includes parts of stable, truncated stable, tempered stable and spectrally positive or negative processes.

■ Stochastic differential equations driven by Lévy processes

We consider the class of driving processes which includes parts of stable, truncated stable, tempered stable and relativistic stable processes. For the stochastic differential equation with such driving processes, we obtained the Hölder conditions on coefficients under which the pathwise uniqueness can be justified, by using Gronwall's inequality.