Research Plan

I will study the following three topics.

[1] The main stream of my study is the Lagrangian mean curvature flow. The goal of this study is as follows. First, introduce a notion of "stability" for Lagrangian submanifolds and define Lagrangian mean curvature flows "with surgery". Next, prove that if the initial Lagrangian submanifold is stable then the Lagrangian mean curvature flow with surgery has a long time solution and converges to a special Lagrangian submanifold. But, the way to reach this goal seems to be vague. So, as steady steps toward the goal, I believe that we have to work on some more concrete problems. So far, it seems to be reasonable to use Bridgeland stability on Fukaya category. Fukaaya category is defined by moduli spaces of holomorphic dicks so that its boundary lives in the given Lagrangian submanifold. Thus, as a first step of this research, I will try to observe the following. (1) When a Lagrangian submanifold moves, how does the moduli space of holomorphic dicks move? (2) When a Lagrangian mean curvature flow develops singularities, how does the moduli space of holomorphic dicks change?

[2] The second topic is a study of nonlinear parabolic partial differential equations. This will be a joint work with Jin Takahashi, a researcher in Tokyo Institute of Technology. He studies nonlinear parabolic PDE from analytic side. Now, we are working on the study of the existence and nonexistence of singular solutions of a certain kind of nonlinear parabolic PDE. Here, a singular solution means it may diverge somewhere called the singular set. We are also interested in asymptotic behavior of the singular solution near the singular set. This research progresses with his analytic techniques and my geometric techniques. So far, the domain where PDEs are defined is complements of higher codimensional submanifolds embedded in a Euclidean space. In the future, we want to develop similar techniques on more general domains including Riemannian manifolds which may be singular. Furthermore, to apply our results to the study of singular Yamabe flows is also a future work.

[3] The third topic is the deformed Hermitian Yang–Mills connections, the so-called dHYM. In 2000, Leung–Yau–Zaslow proved that the special Lagrangian equation and the dHYM equation are transformed into each other, when mirror Calabi–Yau manifolds X and W are given by torus fibrations on some special base manifold. Special Lagrangian submanifolds are studied by many researchers for a long time. In contract, the study of deformed Hermitian Yang–Mills connections are very rare. In 2017, Jacob–Yau introduced the notion of "line bundle mean curvature flow" and proved that the line bundle mean curvature flow converges to a dHYM under some assumption. So far, it is not clear that what happens for general line bundle mean curvature flows when the assumption is dropped. Thus, to deepen this new approach, I will try to research the following. (1) Does a line bundle mean curvature flow blow up in a finite time? When we can not avoid finite time singularities, decide the least blow up rate of the line bundle mean curvature flow. (2) There is Huisken's monotonicity formula for mean curvature flows of submanifolds in Euclidean spaces. Is there an analogous monotonicity formula for line bundle mean curvature flows? (3) There is the notion of "self-similar solutions" for submanifolds in Euclidean spaces. Find a similar notion of "self-similar solutions" for Hermitian connections or Hermitian metrics so that it plays a crucial role to study singularities of line bundle mean curvature flows.