Future research plans (Masataka Iwai)

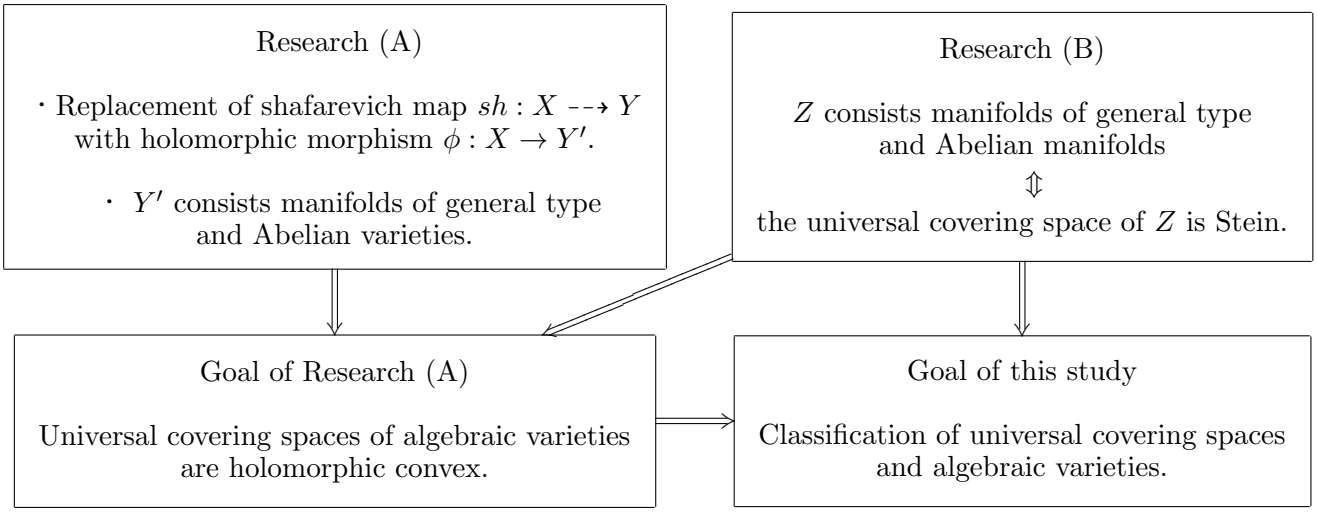
From previous research, the structure of algebraic varieties can be expected to have a simple structure from the viewpoint of universal covering spaces. In future research, we will approach the classification of algebraic varieties by using universal covering spaces. There are an infinite number of complex one-dimensional algebraic varieties, up to biholomorphic equivalences, but their universal covering spaces are limited to three types, such as $\mathbb{CP}^1$, $\mathbb{C}^1$ and an unit ball on $\mathbb{C}^1$. Even in higher dimensions, the universal covering space has a very simple structure and can be expected to be easy to classify. Using complex analysis, we find that the universal covering space is closely related to the original algebraic variety. From the above, it can be expected that examining the classification and structure of the universal covering space will lead to the classification of algebraic varieties.

We perform the following researches for the classification of universal covering spaces and algebraic varieties.

Research (A) Holomorphic convexity of universal covering space of algebraic varieties.

Research (B) On algebraic varieties whose universal covering space is a Stein space.

In Research (A), we investigate that the universal covering space of algebraic varieties is holomorphic convex. A holomorphic convex manifold has an proper morphism with connected fiber to a Stein space (an closed analytic subset of $\mathbb{C}^N$). From research (A), we can expect that the universal covering space of algebraic varieties consists of Stein spaces and algebraic varieties with small dimensions. In Research (B), we study algebraic varieties whose universal covering space is Stein. Researches (A) and (B) greatly contribute to the structure of universal covering spaces and the classification of algebraic varieties.



In **Research (A)**, we study Shafarevich map proposed by Kollár. Shafarevich map is closely related to universal covering spaces and fundamental groups. By further refining the singular foliation theory used in preprint [5], we show that the Shafarevich map can be replaced to a morphism. We show that Y consists manifolds of general type and Abelian manifolds.

In **Research (B)**, we prove that the universal covering space of Z is Stein iff Z consists manifolds of general type and Abelian manifolds. We show that W is a manifold of general type for the Shafarevich map $sh : Z \to W$. We show that there are many holomorphic sections of a pluri-canonical line bundle bundle $K_W^{\otimes m}$. We apply the technique of constructing holomorphic sections by using singular metrics in the paper [1].