Summaries of researches and results (Masataka Iwai)

I outline my paper [1] and my preprints [3][5] on my list of publications. My major is **complex algebraic geometry.** I research about a classification of algebaric varieties (closed sub-manifolds in $\mathbb{CP}^N$) and complex structure by using methods of **algebraic geometry, complex geometry and several complex analysis.**

There is a classification problem: "how many algebraic varieties are there?" Algebraic varieties of two dimensions or less have been classified for many years. For algebraic varieties of three or more dimensions, Mori built a classification algorithm (minimal model theory) and received the Fields Award for his work. However, the classification of algebraic varieties and the construction of moduli of complex structures are not yet complete.

In my paper [1], we studied the following conjecture by Popa-Schnell about a deformation of complex structures.

Conjecture. Let $f : X \to Y$ be a surjective morphism between algebraic varieties, with Y of dimemsion n, and L be an ample line bundle on Y. For any positive integers a, b such that $b \geq a(n+1)$, $f_*(K_X^{\otimes a}) \otimes L^{\otimes b}$ is globally generated.

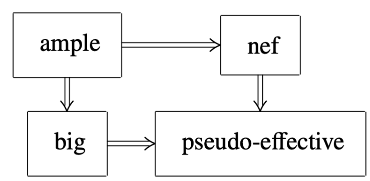
This conjecture is open, even when f is an identity map, and is a very difficult conjecture. Popa-Schnell solved the conjecture under the assumption that L is free. We removed this assumption and partially solved it as follows.

Theorem. For any positive integers a, b such that $b \geq (n^2 - n)/2 + a(n+1)$, the direct image sheaf $f_*(K_X^{\otimes a}) \otimes L^{\otimes b}$ is globally generated on the regular locus of f.

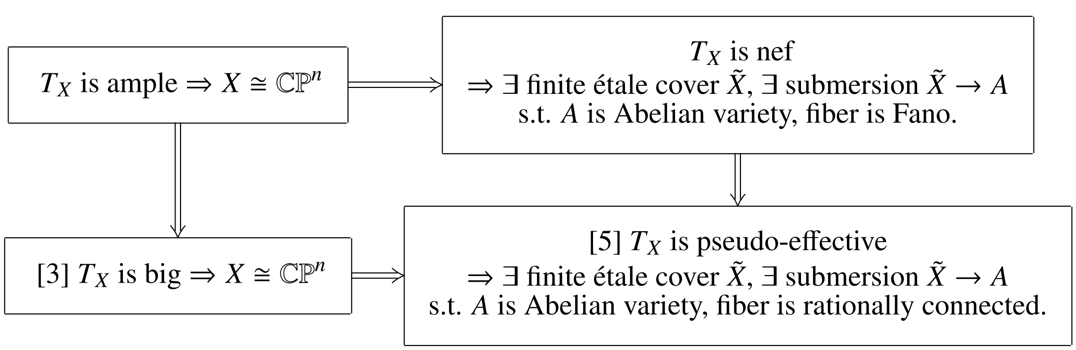
Using the method of previous research, we could not remove the assumption that L is free. For this reason, we used complex analytical methods different from previous studies, such as the injectivity theorem of cohomology for morphisms.

In [3][5], we studied the structure theorem of algebraic varieties whose tangent bundle is big or pseudo-effective.

The most basic concept of algebraic geometry is the ampleness of line bundles. Ampleness is generalized to the notions of nef, big, and pseudo-effective, as shown in the table to the right. These are concepts of algebraic positivity and play a particularly important role in the classification of algebraic varieties in minimal model theory.



Mori showed that if the holomorphic tangent bundle T_X of the algebraic variety X is ample, then X is biholomorphic to $\mathbb{CP}^n$. Campana et al. studied algebraic varieties where T_X is nef. In this way, if the tangent bundle has algebraic positivity, the structure of the algebraic variety is limited. In this study, we studied the structure of algebraic varieties when T_X is big or pseudo-effective. The following is a summary, including previous studies. (Preprint [5] is a collaborative study between Shin-ichi Matsumura and Genki Hosono.)



Fano varieties are known to be rational connected. Thus, these theorems are generalizations of Mori et al.'S work. Since Campana et al.'S work only dealt with smooth metrics, their method was difficult to handle for pseudo-effectiveness. In Preprint [5], we used a completely different method, such as singular metrics and singular foliation structures.