Plans

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(1) Determine a minimal generating set of oriented Roseman moves

A *surface diagram* is the image of a surface-knot via a generic projection from $\mathbb{R}^4$ into $\mathbb{R}^3$, equipped with over/under information. It is known that two surface diagrams are related by a finite sequence of seven types of Roseman moves if and only if they present the same surface-knot. Thus, it is sufficient to show the invariance under seven types of Roseman moves when we construct an invariant of an oriented surface-knot from a surface diagram. Since seven types of Roseman moves are divided into different 50 versions with respect to over/under information and orientations of surfaces, it is hard to check the invariance under all the 50 versions of oriented Roseman moves. Therefore, it is simplified the process to show the invariance if we can find a collection of some oriented Roseman moves which generates all oriented Roseman moves. We call such a collection a *generating set* of oriented Roseman moves. In this study, we attempt to determine the infimum of the number of moves contained in a generating set and to find a minimal generating set.

(2) Develop the augmented Alexander invariant of a surface-knot

Recently, A. Ishii and K. Oshiro are defined the *augmented Alexander invariant* of an oriented knot, which is a generalization of the twisted Alexander invariant and the quandle cocycle invariant. In this study, they and I try to develop the augmented Alexander invariant of an oriented surface-knot. The augmented Alexander invariant of an oriented knot is calculated by constructing a matrix from a diagram of the oriented knot. Now, we similarly construct a matrix from a surface diagram of an oriented surface-knot and investigate its behavior under seven types of Roseman moves. From now on, we define a quantity cancelling change of the matrix under seven types of Roseman moves, and develop the augmented Alexander invariant of an oriented surface-knot. Surface diagrams and their deformations are complicated, we consider oriented sphere-knots or surface diagrams with no branch points if necessary.

(3) Apply the quandle cocycle invariant for a singular surface-knot

The *quandle cocycle invariant* is an invariant of a surface-knot obtained from a 3-cocycle of a quandle (co)homology group, and it is applied to detection of the non-invertibility and estimation of the triple point number. It is known that it does not usually becomes an invariant of a singular surface-knot, where a *singular surface-knot* is a surface-knot admitting transverse double points. I have already modified a quandle (co)homology group so that it induces the quandle cocycle invariant for a singular surface-knot. In this study, we find concrete examples of 3-cocycles of the modified quandle (co)homology groups using computer and to apply them to detection of the non-invertibility and estimation of the triple point number.