(2) Study proposal Makiko Mase

(i) Aims of the study

The aim of the study is to understand algebro-geometric aspects of algebraic complex $K3$ surfaces, which we simply call a $K3$ surface. It is known that geometry of $K3$ surfaces is related to singularity theory, symplectic geometry, and the homological mirror conjecture. Not individually, we are interested in a relation between $K3$ surfaces in a projective ambient space (possibly singular) and deformation of singularities of the ambient space. It is important to characterize $K3$ surfaces from a view-point of algebraic curves in it, and from maps from a $K3$ surface to a Lie algebra. We expect to understand a degeneration of $K3$ surfaces in a family and to construct some "nice" compactification of the moduli.

Problems

1. The deformation of singularities and $K3$ surfaces.
2. The moduli space of maps from a $K3$ surface to a Lie algebra.
3. Weierstrass semi-groups of pointed curves in a $K3$ surface.
4. A degeneration of $K3$ surfaces in a family and the compcatification of the moduli.

(ii) Study methods

Problem 1 The deformation of a singularity produces the Milnor lattice together with the monodromy action on it. In a joint work with Professor Claus Hertling in the University of Mannheim, we study an algebraic structure of the Milnor lattice of isolated hypersurface singularities in a relation with the Orlik conjecture. And then, we intend to study a Torelli-type theorem for the Milnor lattice to understand the Frobenius structure of the base space of unfolding for singularities. In the special case of dimension 3, we would like to apply the study to characterize algebraic $K3$ surfaces.

Problem 2 Considering the definition, there naturally exists a map from an elliptic $K3$ surface to a Lie group. Being related to harmonic maps, it is important to study maps from a manifold to a Lie group in a prospect to differential geometry and differntial equations. We shall first characterise the moduli space of holomorphic maps from a $K3$ surface to a Grassmann variety, and then, we extend this to more general cases.

Problem 3 This is a joint work with Professor Jiryo Komeda in Kanagawa Institute of Technology. For a given semigroup, it is important to find out a pointed curve that admit the semigroup as its Weierstrass semigroup.

An interesting construction of a $K3$ surface is via a double covering of a rational elliptic surface. Any rational elliptic surface is obtained by blowing up nine points in the projective plane, and all the rational elliptic surfaces are classified by describing their singular fibres. We intend to give curves in a $K3$ surface that admit a given Weierstrass semigroup.

Problem 4 There are many ways of degeneration of algebraic surfaces: for instance, the GIT quotient due to Mumford. On the other hand, the compactification of the moduli space is one of major problems in algebraic geometry. In the study, we would like to describe some nice degeneration of $K3$ surfaces, in particular, for $K3$ surfaces in the weighted projective spaces, where $K3$ surfaces are obtained as anticanonical sections of the space.

(iii) Aspects

We expect that we will be able to characterise $K3$ surfaces by giving explicit subvarieties, and describe degeneration in a family. That will be applied to construct a moduli space together with its compactification with a relation of some deformation of singularities in the ambient space.