(2) Abstract of results Makiko Mase

The applicant has studied dualities of polytopes and of Picard lattices between families of weighted $K3$ surfaces associated to isolated hypersurface singularities in $\mathbb{C}^3$.

Investigated by Arnold, unimodal singularities admit a strange duality, which is related by Pinkham with a duality of lattices of $K3$ surfaces. Extended for invertible polynomials by Ebeling and Takahashi, the strange duality for singularities is our motivation.

We focus on the strange duality for bimodal singularities, which are defined by weighted homogeneous invertible polynomials, and are projecitivized as anticanonical sections in a weighted projective space with weight system determining simple $K3$ singularities classified by Yonemura. We have seen that if the projectivizations are invertible, then, the families of $K3$ surfaces admit the polytope duality, a part of which furthermore admit the duality of Picard lattices.

The following three papers under submission are motivated by questions set by Professor Wolfgang Ebeling.

(a) **Polytope duality for families of $K3$ surfaces associated to Q_{16} and S_{16}.**

The aim is to understand a strange duality for bimodal singularities of types Q_{16} and S_{16}, which we have omitted since the projectivization is not invertible.

Let f be a defining polynomial of a singularity of type Q_{16} or S_{16}. According to a study by Ebeling and Ploog, the singularity $(f = 0)$ is strange dual to itself, and admits a projctivization F that is not an invertible polynomial. Unfortunately, it is impossible to set the same question as in a series of studies by Mase and Ueda, and Mase for strange-dual pairs of bimodal singularities that admit invertible projectivization. However, it is possible to consider the Newton polytope of F as a subpolytope of the polytope of the weighted projective space $\mathbb{P}_a$ with weight a of which the polynomial F defines an anticanonical section. Under this situation, denote by Δ_F the Newton polytope of F, and Δ_a the polytope determining the space $\mathbb{P}_a$. We consider whether or not there exists a reflexive polytope $\Delta \subset \Delta_a$ such that Δ_F is a subpolytope of Δ and that the polar dual Δ^* coincides with Δ. As a result, we obtained a negative answer to the question.

(b) **Polytope duality for families of $K3$ surfaces and coupling.**

Introduced by Ebeling, coupling is a duality between two weight systems using the magic square. Bisides, for some weight systems that are coupling pairs, one can find a pair of defining invertible polynomials of some singularities in $\mathbb{C}^3$ to be strange-dual, and then, they can be projectivized as anticanonical sections in the weighted projective spaces with weight systems determining simple $K3$ singularities.

Pick up defining polynomials f and f' of singularities in $\mathbb{C}^3$ that are strange dual as are constructed explained. Take their invertible projectivizations F and F' respectively as anticanonical sections in the weighted projective spaces $\mathbb{P}_a$ and $\mathbb{P}_b$ of weights a and b. We ask a question if there exists a pair of reflexive polytopes Δ and Δ' such that $\Delta_F \subset \Delta \subset \Delta_a$, $\Delta_{F'} \subset \Delta' \subset \Delta_b$, and $\Delta^* \simeq \Delta$ hold. We concluded that almost all the strange-dual pairs admit such a pair of polytopes.

(c) **Lattice duality for coupling pairs admitting polytope duality with $L_0(\Delta) = 0$.**

We consider polytope-dual pairs obtained in part (b) with the toric contribution $L_0(\Delta)$ is trivial, in which case the ambient space, associated to the polytope Δ, has at most simplicial singularities, and the Picard lattice of the family of $K3$ surfaces are simply the restriction of the Picard group of the ambient space. In this article, we conjectured that these polytope-dual pairs extend to a duality of lattices. We concluded that our conjecture is true for all pairs, and give an explicit form of Picard lattices for each lattice-dual pairs.