これまでの研究成果（英訳）

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So far I have studied several complex variables. In particular, I got results on

- **(1):** automorphism groups on unbounded homogeneous domains
- **(2):** a characterization of homogeneous domains by their automorphism groups
- **(3):** the existence or non-existence of co-compact discrete subgroups in automorphism groups
- **(4):** the Lie group structures of automorphism groups
- **(5):** the extension of holomorphic functions using L^2 $\bar{\partial}$ estimates
- **(6):** analysis of multiple zeta functions.

In the above subjects, mainly I have studied on the automorphism groups of domains in complex Euclidean spaces. This point will be described since it is related to future research plans.

We denote by $\mathrm{Aut}(M)$ the automorphism group of a complex manifold M, which is a set of all biholomorphic self-mappings. $\mathrm{Aut}(M)$ is an invariant object by holomorphic transformations and used in complex geometry. $\mathrm{Aut}(M)$ has a natural structure of a topological group and, in particular, if M is a bounded domain of a Euclidean space, $\mathrm{Aut}(M)$ becomes a Lie group. However, in general, $\mathrm{Aut}(M)$ is not a Lie group for some complex manifold M, and so it is asked whether $\mathrm{Aut}(M)$ has a Lie group structure for a given manifold M (4). Determining the structure of $\mathrm{Aut}(M)$ for a given manifold M is an interesting problem, and in particular, it is important when M is a homogeneous space. In the case of bounded domains, there is a theory showing the correspondence between Lie algebras of automorphism groups and the bounded domains. There is no such correspondence in the case of unbounded domains. To find some kind of association, I have tried to understand the structure of automorphism groups even in the case of unbounded domains (1). The important notion of the theory of geometry are the homogeneous space and their quotient spaces. In the case of complex manifolds, study of automorphism groups leads to study of quotient space (3). Automorphism groups are invariant by holomorphic transformations. It means that, for complex manifolds M, N, if $M \simeq N$ holds as complex manifolds, then $\mathrm{Aut}(M) \simeq \mathrm{Aut}(N)$ holds as topological groups. The characterization of M by the automorphism group means the opposite of this property. Namely if $\mathrm{Aut}(M) \simeq \mathrm{Aut}(N)$ as topological groups, then $M \simeq N$ holds as complex manifolds. This is not always the case. We thought that, for homogeneous spaces, the characterization would hold since automorphism groups are large (2). However, we got a counterexample contrary to the expectation in our paper.

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We describe the results of (2) and (3). We consider the following domains:

$$D^{p,q} = \{(z_1, \dots, z_{p+q}) \in \mathbb{C}^{p+q} : -|z_1|^2 \cdots - |z_q|^2 + |z_{q+1}|^2 + \cdots + |z_{p+q}|^2 > 0\}.$$

Here, p, q are positive integers. The domains are homogeneous. The reason to study such domains comes from study of the hypersurfaces

$$S^{p,q} = \{(x_1, \dots, x_{p+q}) \in \mathbb{R}^{p+q} : -x_1^2 \cdots - x_q^2 + x_{q+1}^2 + \cdots + x_{p+q}^2 = 1\}$$

and their quotients by discrete groups. Calabi-Markus and Wolf showed that any discrete subgroup of the isometric group of $S^{p,q}$, $p > q$ acting properly discontinuously is finite. Therefore the quotient space of the subgroup is not compact. This is a decisive difference compared to the hyperbolic space $S^{1,q}$. One of the motivations of the research was what would happen in the complex domain $D^{p,q}$ of such a phenomenon. For this purpose, we have determined the automorphism group of $\mathrm{Aut}(D^{p,q})$. For example, $\mathrm{Aut}(D^{p,q}) = U(p,q) \times \mathbb{R}_{>0}$ if $p > 1$. Then, we showed that the quotient spaces of $D^{p,q}$, $p > q$, is not compact. Also, by classifying complex manifolds on which $U(p,q) \times \mathbb{R}_{>0}$ acts, we determined when $D^{p,q}$ is characterized by its automorphism group and when it is not. There had been no known example in which this characterization does not hold for homogeneous domains, and the example became the first counterexample.