

Results of research

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We have been studying the properties of the matrix models which are closely related to the supersymmetric gauge theories and obtained the following results.

1. We consider β -deformed quiver matrix model based on the affine Lie algebra $A_n^{(1)}$. The Virasoro constraint of this model is determined. For $n = 1, 2$ cases, the explicit forms of the loop equations are obtained (Ref. [30] of the Publication List).
2. The (W)AGT conjecture implies there is a correspondence between the partition functions of the four-dimensional $\mathcal{N} = 2$ supersymmetric gauge theories and the conformal blocks of the two-dimensional theories with the Virasoro or W symmetries. The “ q -deformed” version of (W)AGT conjecture states that the q -lifted version of the partition function of five-dimensional gauge theories and the “conformal blocks” of the two-dimensional theories with the q -deformed Virasoro/ W symmetries. Starting from this q -version of (W)AGT conjecture, we demonstrate by taking a certain r -th root of unity limit in q , the correspondence between the four-dimensional partition function on the ALE space of A-type and the conformal blocks of the two-dimensional theories with the super-Virasoro symmetry or its generalization is automatically generated ([32]). Furthermore, we demonstrated that the parafermions appear in the r -th root of limit of the q -deformed Virasoro and the q -deformed W algebra ([34]).
3. A q -deformed vertex operator is determined from the five-dimensional $SU(2)$ Nekrasov partition function based on the q -AGT conjecture. We obtained a q -deformed version of Coulomb gas representation of the conformal block by using the vertex operators and q -screening charges. After slightly changing the position of one of the vertex operators, we have checked that the q -block coincides with 5D Nekrasov function in low degrees of instanton expansion ([35]).
4. A set of Schwinger-Dyson equations for the resolvents are considered in a class of supersymmetric Chern-Simons-matter matrix models. In the planar limit, these loop equations reduce to two independent algebraic cubic equations for the two planar resolvents ([36]).
5. We argue that the level-1 elliptic algebra $U_{q,p}(\widehat{\mathfrak{g}})$ is a dynamical symmetry in the correspondence between 2d field theories and 5d supersymmetric gauge theories. A level-1 $U_{q,p}(\widehat{\mathfrak{sl}}(2))$ module can be realized by an elliptic version of the Frenkel-Kac construction. In a r -th root of unity limit of the deformation parameter p , the \mathbb{Z}_r -parafermions and a free boson appear. And the 2d/5d correspondence goes to the correspondence between the 2d coset CFT with para-Virasoro symmetry and 4d $\mathcal{N} = 2$ $SU(2)$ gauge theory on $\mathbb{R}^4/\mathbb{Z}_r$.
6. We have argued that a unitary matrix model, which is an extension of the Gross-Witten-Wadia model by the logarithmic potential, is closely related to the $\mathcal{N} = 2$ supersymmetric $SU(2)$ gauge theory with two matter hypermultiplets. The spectral curve of the matrix model is isomorphic to the Seiberg-Witten curve of the gauge theory. Using the method of orthogonal polynomials, it is shown that the partition function is a tau function of the Painlevé equation. The double scaling limit of the matrix model corresponds to the limit of the gauge theory to the Argyres-Douglas superconformal fixed point ([38, 39]).