Plan

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The followings are the plan of study related to the Chern-Simons perturbation theory.

１．I will revisit to the construction of the higher degree term of the Bott-Cattaneo Chern-Simons perturbation theory to remove the gap by using the technic I developed in [1] with A. S. Cattaneo.

２．I will study the U(1)(or SO(2))-Chern-Simons perturbation theory defined by using a similar way as the Bott-Cattaneo construction. In particular, I will compare it with Lescop's invariant and Watanabe's invariant of 3-manifolds with $b_1=1$.

３．I will study a cohomology class corresponding to the "gap" on the original Bott-Cattaneo construction. The gap can be quantified as a cohomology class of the second cohomology group of the given manifold with a local system coefficient. A similar cohomology class appears in a construction of Lescop's invariant of 3-manifolds with $b_1=1$. It is known that Lescop's cohomology class is deeply related to the Alexander polynomial of the manifold. I will investigate the topological meaning of these cohomology class.

４．I will investigate a behaviour of the Chern-Simons perturbation theory under the appropriate surgeries among 3-manifolds with local systems (surgery formulas).

5．I will study the relationship between SU(2)-Chern-Simons perturbation theory and the Casson invariant. The Casson invariant is given as an algebraic number of irreducible representations of the fundamental group. On the other hand, for each irreducible representations, the SU(2)-Chern-Simons perturbation theory at the corresponding local system of it gives a real value. So it is expected that SU(2)-Chern-Simons perturbation theory gives a refinement of the Casson invariant.

6. I will study the Casson invariant and the Chern-Simons perturbation theory from the point of view of the singularity theory of stable maps.

References

[1] Alberto S. Cattaneo and Tatsuro Shimizu, A note on the Θ-invariant of 3-manifolds, arXiv:1903.04386, 2019