Research results

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• On the braid index of Kanenobu knots

Every knot is presented as a closed braid. The braid index of a knot is the minimum number of strings of a braid needed for the knot to be presented as a closed braid. The MFW inequality gives a lower bound of the braid index of a knot by applying the v-span of the HOMFLYPT polynomial. Since Kanenobu knots $k(n)$ ($n = 0, 1, 2, \dots$) have the same HOMFLYPT polynomial, it is not easy to determine the braid index $\beta(k(n))$ of $k(n)$. We give a sharper lower bound of $\beta(k(n))$ by applying the $(2, q)$-cabling of the Γ-polynomial.

• On the arc index of Kanenobu knots (Joint work with Hwa Jeong Lee)

Every knot has an arc presentation. The arc index of a knot is the minimum number of pages needed for the knot to be presented as an arc presentation. The MB inequality gives a lower bound of the arc index of a knot by applying the a-span of the Kauffman polynomial. Since Kanenobu knots $k(n)$ ($n = 0, 1, 2, \dots$) have the same a-span of the Kauffman polynomials, it is not easy to determine the arc index $\alpha(k(n))$ of $k(n)$. We construct "canonical cabling algorithm" which gives sharper upper bounds of the arc index of cable knots and give a sharper lower bound of $\alpha(k(n))$ by applying "canonical cabling algorithm" and the $(2, q)$-cabling of the Γ-polynomial.

• The (p, q)-cabling of the Γ-polynomial for mutant knots

A mutant knot is a possibly different knot obtained from a knot by an operation called mutation. It is known that many knot invariants are invariant under mutation, for example, the HOMFLYPT and Kauffman polynomials, and their $(2, q)$-cablings are invariant under mutation. On the other hand, it is known that the $(3, q)$-cabling of the HOMFLYPT polynomial distinguishes a mutant knot pair. We show that the $(3, q)$-cabling of the Γ-polynomial is invariant under mutation. (Recently, Tetsuya Ito showed that the (p, q)-cabling of the Γ-polynomial is invariant under mutation for any coprime integers $p(> 0)$ and q.)

• A characterization of the Γ-polynomials of knots with clasp number at most two

Every knot bounds a singular disk with only clasp singularities, which is called a clasp disk. The clasp number of a knot is the minimum number of clasp singularities among all clasp disks of the knot. It is known that the Conway polynomials of knots with clasp number at most two are characterized. We characterize the Γ-polynomials of knots with clasp number at most two.

• Studies on the $(2, 1)$-cabling of the Γ-polynomial

Since it is known that the Γ-polynomial is computable in polynomial time, the (p, q)-cabling of the Γ-polynomial is also computable in polynomial time. We show that the $(2, 1)$-cabling of the Γ-polynomial completely classifies the unoriented knots with up to ten crossings including the chirality information. Moreover, we show that there exist infinitely many knots with the trivial $(2, 1)$-cabling of the Γ-polynomial. Furthermore, we see that the knots have the trivial Γ-polynomial, the trivial first coefficient HOMFLYPT and Kauffman polynomials.

• The self-smoothing number of knots and links

We call smoothing a self-crossing point of an oriented link diagram self-smoothing. By self-smoothing repeatedly, we obtain an oriented link diagram without self-crossing points. We show that every knot has an oriented diagram which becomes a two-component oriented link diagram without self-crossing points by a single self-smoothing.

• Classification of Abe-Tange's ribbon knots

Abe and Tange constructed a sequence of slice disks with the same exterior. Moreover, they showed that these slice disks are ribbon disks. We call the boundaries of the ribbon disks Abe-Tange's ribbon knots. We classify Abe-Tange's ribbon knots completely by using the Γ-polynomial.