# Study Plan

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Based on my previous studies, I will study the following topic. (Hereafter, the citation numbers of papers written below denote the corresponding numbers in the Paper List.)

**The time periodic problem of the incompressible Navier-Stokes equations on exterior domain with moving boundary**

We can find that when a tale of a fish moves periodically in water, a time periodic flow appears around the tale. When a water balloon oscillates periodically, a time periodic flow also appears in the balloon. When we consider to analyze these phenomena strictly in mathematical sense, the analysis is located in time periodic problems of the incompressible Navier-Stokes equations with periodically moving boundary of domains. The former is the problem on an exterior domain, while the latter is that on a bounded domain. These problems appear as very natural phenomena in our life and the mathematical analysis is thus important, however, the study has many unsolved problems. For example, **no research about exterior domain case exits as far as we investigated.** Hence, I will challenge the time periodic problem in exterior domain case based on my previous study about bounded domain case (Paper [2-1]).

The key of proofs in Paper [2-1] is constructing time decay estimates of solution operator due to deriving a uniform resolvent estimate. In time periodic problems, generally speaking, it is important to derive time decay estimates of solution operator by properties of the Poincaré mapping. In previous works, Miyakawa and Teramoto (1983) study the time periodic problem by constructing weak solutions, while Saal (2006) studies the initial value problem by using maximal regularity theory. However, these results do not derive the time decay estimates. **When we reformulate the moving boundary problem to a problem on a fixed domain, we have to handle abstract parabolic system whose linear operator has perturbation terms whose coefficients depend on time variable, and the Helmholtz projection which is a non-local operator and depends on time variable. Therefore deriving the decay estimate is more difficult.** In fact, the solution operator consists of a sum of a term of the semi-group and that defined by an inductive operation of operators. Even analysis for the semi-group part, Saal (2006) shows some resolvent estimate, however, the constants in the estimate depend on time variable because the linear operator has coefficients depending on time. In Paper [2-1], we succeeded to derive a uniform resolvent estimate on time by introducing the Taylor expansion of operators. By applying the estimate, we also succeeded to derive the time decay estimates of the solution operator and we proved existence of time periodic solutions. **Note that the resolvent estimate is applicable to the exterior domain case.**

I consider to show existence of time periodic solutions for the exterior domain case with moving boundary by the Poincaré mapping and deriving time decay estimates of solution operator based on the resolvent estimate. Since we assume a situation such that the motion of boundary is very small, I think that the result by Yamazaki (2000), which handles a usual fixed boundary case, is useful for deriving the decay estimate, that is, estimates corresponding to the results in the Yamazaki paper can be established. In Yamazaki (2000), the key idea of proof is Strichartz type estimates of linear operator on Lorentz spaces. Hence I will try to construct similar type estimates to the Strichartz estimates and to derive the time decay estimate of the solution operator. If we obtain time periodic solutions, I will also try to show regularity of the periodic solution.