About my study Kazuyuki Tsuda (Graduate School of Engineering Science, Osaka University)

I have studied the mathematical analysis of nonlinear partial differencial equations. Especially, I have studied fundamental equations of the fluid dynamics, the compressible and incompressible Navier-Stokes equations. These systems have given us many basic problems about existence, uniqueness, regularity, stability and asymptotic behavior of solutions. They have deep mathematical structures and have been applied in the physics and engineering science. Navier-Stokes equations was first founded by Navier, which was an engineer in France, in 1827, and has been studied since the year. However, since the equations have profoundness of mathematical theory, there are still many unsolved mathematical problems including the millennium prize problems provided by the Clay Mathematics Institute and thus many researchers have studied the system.

Under the background, I have mainly studied time periodic problem of the compressible and incompressible Navier-Stokes equations and analysis for asymptotic behavior of solutions to the compressible Navier-Stokes-Korteweg system as time goes to infinity. (Hereafter, the citation numbers of papers written below denote the corresponding numbers in the Paper List.)

(1) The time periodic flow is fundamental phenomena in motions of fluid and thus many researchers have studied the flow, but the mathematical analysis is in general more difficult than that around a constant state of the system. In fact, if we consider the stability analysis of time periodic flows or time periodic flows which appear from periodic oscillations of boundary of domains, there are many unsolved mathematical problems because linear operators of the systems which dominate behavior of solutions have variable coefficients and we thus have difficulties for analysis of the linear operators. I succeeded to show the existence and stability of time periodic solutions to the compressible Navier-Stokes equations in the three dimensional whole space which had been an open problem in long time. (Paper [1-2], Paper [1-1] is a related study.) Furthermore, I partly solved the time problem in the two dimensional case. (Paper [1-4]). I also study the incompressible Navier-Stokes equation with periodically moving boundary and showed existence of time periodic solutions in L^p space including the scale invariant L^n space. (n denotes the space dimension.) (Paper [2-1])

(2) The Navier-Stokes-Korteweg system describes motion of liquid-vapor type two phase flows with phase transitions by using the diffuse interface. A value of derivative of pressure, which is one of elements dominating the phase transition, characterizes the system in mathematical sense. If the value is positive, the system is in a single phase, and is a quasi-linear hyperbolic-parabolic system. Behaviors of solutions have not only influences of the viscosity terms in the system, that is, parabolic aspect of the system, but also wave properties of fluid, that is, hyperbolic aspect of the system. Interaction between the parabolic and hyperbolic aspects causes more complicated behaviors of solution. On the other hand, when the phase transition appears, the sign of the value of derivative of pressure is changed and the hyperbolic aspect is lost. When the value is positive, I showed diffusion wave properties which have both properties of the parabolic aspect and hyperbolic aspect of the system in the leading parts of solutions as time goes to infinity. (Paper [1-5]) I also showed that if we take some special initial data in the two dimensional case, the behavior of the diffusion wave has big difference in contrast to usual case. (Paper[2-3])

Furthermore, when the value of derivative of pressure is equal to 0, that is, the hyperbolic aspect of the system degenerates, we revealed asymptotic behaviors of solutions in the critical case. (Paper[1-9]).

Other results

I had joined a project entitled "The development of innovative energy devices-creation and implementation of nano composite dielectric material" by Ministry of Education, Culture, Sports, Science and Technology as a post doctoral fellow. In the project, I studied mathematical theories of controls for a parallel-wire driven robot with an active balancer and shape memory alloys. (The purposes of the studies are to show mathematically that error from a given desired orbit converges to 0 in long time when suitable control laws are inputted to the system by using the dynamics of the systems and control theories. This is a kind of stability theories.) The results are published in Paper [1-5,1-6,1-8].