# Research Results

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I am working on discretization of surfaces based on integrable systems techniques. In particular, I am interested in discretization of linear Weingarten surfaces. So far I have shown several results as follows (references can be found in "List of Papers"):

**1. Research on semi-discrete minimal surfaces in Euclidean 3-space ([1])**

With Wayne Rossman, we gave a Weierstrass representation for semi-discrete minimal surfaces and constructed new semi-discrete minimal surfaces. As an application, we discretized catenoid via various methods, and we showed that semi-discrete minimal catenoids in the sense of Müller, Wallner have the same profile curves as both discrete and smooth catenoids. This indicates that semi-discrete surfaces help us to recognize the similarities and differences between smooth and discrete surfaces.

**2. Research on discretized maximal surfaces in Minkowski 3-space ([2], [9], [11])**

We discretized maximal surfaces as a special class in isothermic surfaces in Minkowski 3-space. We first gave Weierstrass representations for discretized maximal surfaces, and described their singularities. Singularities of discretized surfaces had not been explicitly defined before this. Based on these results, we aim to describe singularities of general discretized surfaces.

Furthermore, with Wai Yeung Lam, we described trivalent maximal surfaces in Minkowski 3-space. It was shown that trivalent maximal surfaces could be constructed via a Weierstrass-type representation. Moreover, in this case, it was shown that their associated family could be also treated. We characterized singularities of associated families of trivalent maximal surfaces. This is the first step toward a discrete surface theory independent of the choice of coordinate systems.

**3. Research on discrete linear Weingarten surfaces in 3-dimensional spaceforms ([5], [10])**

With Wayne Rossman, we analyzed the discrete linear Weingarten surfaces of Bryant type (BrLW) in hyperbolic 3-space. BrLW surfaces are linear Weingarten surfaces which lie in a particular deformation family between flat surfaces and discrete constant mean curvature (CMC) 1 surfaces. We considered singularities of discrete BrLW and their unit normal vector fields in de Sitter 3-space. In particular, in a certain situation, we showed that the condition for singularities of discrete CMC 1 surfaces in de Sitter 3-space to appear is the same as the one for discrete maximal surfaces.

**4. Research on discrete CMC surfaces in 3-dimensional Riemannian spaceforms ([8])**

Hoffmann introduced a construction of discrete non-zero constant mean curvature surfaces in Euclidean 3-space using matrix factorizations. However, there are several gaps in the proofs, so it is not clear that any discrete constant mean curvature surface can be constructed via his method. With Yuta Ogata, we completely filled the gaps and we extended the construction to discrete constant mean curvature surfaces in 3-dimensional Riemannian spaceforms. This construction gives a geometric solution of a discrete version of a sinh-Gordon equation proposed by Pedit, Wu. Moreover, we analyzed singularities of discrete constant positive Gaussian curvature surfaces obtained by parallel surfaces of discrete constant mean curvature surfaces. This result is highly related to the analysis of discrete sinh-Gordon equation.

In addition, we classified semi-discrete surfaces of revolution in Euclidean and Minkowski 3-spaces, derived Weierstrass-type representations for smooth timelike constant mean curvature surfaces in Minkowski and AdS 3-spaces and analyzed their singularities, and proposed a new theory of discrete timelike surfaces ([7]). Furthermore, we had written a book about recent progress on our discrete surface theory ([13]).