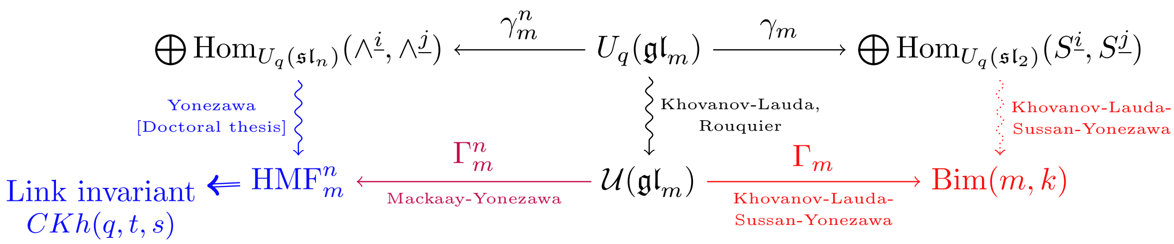
M. Khovanov constructed a homological link invariant which is a refinement of the Jones polynomial. As well-known, the Jones polynomial is a quantum link invariant defined using a quantum group $U_q(\mathfrak{sl}_2)$ and its vector representation. From this fact, I have been working on a natural question:

Can we construct homological link invariants which refine other quantum link invariants?



(1) Summary of the paper "Quantum $(\mathfrak{sl}_n, \wedge V_n)$ link invariant and matrix factorizations": Khovanov and Rozansky introduced matrix factorizations defining a homological link invariant which refines the quantum link invariant constructed using $U_q(\mathfrak{sl}_n)$ and its vector representation. In this paper, we generalize Khovanov–Rozansky's matrix factorizations and define a new link invariant $CKh(q,t,s)$ which refines the quantum link invariant $CJ(q)$ constructed using $U_q(\mathfrak{sl}_n)$ and its fundamental representations (Blue text study in the above figure). The link invariant $CJ(q)$ is recovered as $CKh(q,-1,1)$.

(2) Summary of the thesis "$\mathfrak{sl}_N$-Web categories and categorified skew Howe duality": On the antisymmetric tensor $\wedge^k(\mathbb{C}^n \otimes \mathbb{C}^m)$, there exist a left $U_q(\mathfrak{sl}_n)$ action and a right $U_q(\mathfrak{gl}_m)$ action such that these two actions commute. So we have a $U_q(\mathfrak{gl}_m)$ representation (left-up morphism in the above figure).

$$\gamma_m^n : U_q(\mathfrak{gl}_m) \to \bigoplus_{\sum_{\alpha=1}^m i_\alpha = k,\, \sum_{\alpha=1}^m j_\alpha = k} \mathrm{Hom}_{U_q(\mathfrak{sl}_n)}(\wedge^{i_1} \otimes \cdots \otimes \wedge^{i_m}, \wedge^{j_1} \otimes \cdots \otimes \wedge^{j_m}),$$

where $\wedge^i$ is the i-th fundamental representation of $U_q(\mathfrak{sl}_n)$ $(i = 1, ..., n-1)$ and the trivial representation $(i = 0, n)$. We have two facts: (A) The quantum group $U_q(\mathfrak{gl}_m)$ is categorified by the category $\mathcal{U}(\mathfrak{gl}_m)$ introduced by Khovanov–Lauda and Rouquier(center wavy arrow in the above figure) and (B) $\bigoplus \mathrm{Hom}_{U_q(\mathfrak{sl}_n)}(\wedge^{\underline{i}}, \wedge^{\underline{j}})$ is categorified by the category of matrix factorizations HMF_n^m in my thesis (left wavy arrow in the above figure). From these facts, we expected that there exists a functor $\Gamma_m^n : \mathcal{U}(\mathfrak{gl}_m) \to \mathrm{HMF}_m^n$ (left-down red functor in the above figure) and we constructed the functor in this paper.

(3) Summary of the paper "Braid group actions from categorical symmetric Howe duality on deformed Webster algebras": On the symmetric product $S^k(\mathbb{C}^2 \otimes \mathbb{C}^m)$, we have a $U_q(\mathfrak{gl}_m)$ representation (right-up morphism in the above figure). From the fact that the tensor representation $S^{\underline{i}} = V_{i_1\varpi} \otimes \cdots \otimes V_{i_m\varpi}$ is categorified by the projective module category of the Webster algebra, we expected that there exists a functor from the category $\mathcal{U}(\mathfrak{gl}_m)$ to the bimodule category of the Webster algebra. However, there are extra complications using the original Webster algebra. In this paper, we defined a deformed Webster algebra $W(\mathbf{s},k)$ and constructed a functor Γ_m from $\mathcal{U}(\mathfrak{gl}_m)$ to the bimodule category $\mathrm{Bim}(m,k)$ of $W(\mathbf{s},k)$ (right-down orange functor). Subsequently, we defined a braid group action on the homotopy category $K^b(\mathrm{Bim}(m,k))$ using the functor.