Four Dimensional Topology

November 29 – December 1, 2019
Room E404/406/408, Department of Mathematics, Osaka University

Abstracts

Kouki Sato (University of Tokyo)
Category of formal knot complexes

Formal knot complex is an algebraic generalization of the knot Floer complex called CFK^∞. We prove that the set of certain stable homotopy equivalence classes of formal knot complexes forms an abelian group (called the formal knot concordance group), where the map from knots in S^3 to their CFK^∞ induces a group homomorphism from the knot concordance group to the formal knot concordance group. Moreover, we introduce an infinite family $\{G_n\}$ of invariants of formal knot complexes under stable homotopy equivalence, which gives an infinite family of new knot concordance invariants. In particular, the primary invariant G_0 determines the concordance invariants called τ, V_k, ν^+ and Υ. By using $\{G_n\}$, we prove that there exist infinitely many formal knot complexes with genus one which cannot be realized by any knot in S^3.

Masaki Taniguchi (University of Tokyo)
Homology cobordism group of homology 3-spheres and Chern-Simons functional
(joint work with Yuta Nozaki and Kouki Sato)

We give a new family of real-valued invariants $\{r_s(Y)\}$ of oriented homology 3-spheres. These invariants are defined by using a filtered version of instanton Floer homology and are closely related to the existence of solutions to ASD equations on $Y \times \mathbb{R}$ for a given homology sphere Y. We have shown several properties of $\{r_s(Y)\}$ including a connected sum formula and a negative definite inequality. In this talk, using the properties of $\{r_s(Y)\}$, we introduce a pseudo metric on the kernel of Froyshov homomorphism in the homology cobordism group. This is joint work with Yuta Nozaki and Kouki Sato.

Shigeru Takamura (Kyoto University)
New geometry behind group actions

We develop a new geometry behind group actions (even on sets), to treat group actions as if they were 'spaces'. The details will appear in S. Takamura "*Actional Geometry*" Lecture Notes. This geometry is *in spirit* close to Grothendieck's scheme theory, but far different from transformation group theory, geometric invariant theory, and stack theory. To treat group actions as if they were 'spaces', *what should be 'topologies' on group actions?*, *what should be globalizations of group actions* (*when group actions are regarded as local objects*)*?* The 'topologies' we introduce on group actions are "pointless topologies" defined via the *higher Galois correspondences* between the irredundant generating systems of subgroups and the intersections of fixed point sets. A group action with this 'topology' is a *cadaster*. For cadasters, in terms of their 'topologies' we define sheaves, cohomologies, and fundamental groups — group actions are treated as 'spaces', and a patching of group actions is formulated as a patching of cadasters. For the mapping class group action on a Teichmüller space, its cadaster reflects how, in the moduli space, the strata of Riemann surfaces with the same automorphism groups intersect.

Mizuki Fukuda (Tokyo Gakugei University)

On $\Gamma(p, q, r)$-representations of branched twist spins

A branched twist spin is a fibered 2-knot in S^4 whose fiber is a cyclic branched cover of S^3 along K. It is known that a presentation of the knot group of the branched twist spin is obtained from that of K or that of the fiber of branched twist spin, and it always has a non-trivial center. A binary triangle group $Gamma(p, q, r)$ has a non-trivial center and the quotient of $Gamma(p, q, r)$ by the subgroup generated by the center is a triangle group. In this talk, we determine the number of $Gamma(p, q, r)$-representaions on a branched twist spin for $Gamma(p, q, r)$ with spherical case.

Kokoro Tanaka (Tokyo Gakugei University)

The bridge number of surface links and kei colorings

(joint work with Kouki Sato)

A bridge trisection is a relative notion of a trisection for surface links in the 4-sphere. The bridge number of a surface link is the minimal number of bridges required in all the possible bridge trisections of the surface link. In this talk, we give lower bounds of the bridge numbers of surface links by using kei colorings. As an application, we show that there exists a surface knot with bridge number k for any positive integer k.

Kenta Hayano (Keio University)

Classification of genus-1 holomorphic Lefschetz pencils

(joint work with Noriyuki Hamada)

It is well-known that generic pencils in the linear systems $|3H|$ and $|2C + 2F|$ are genus-1 holomorphic Lefschetz pencils, where H is a projective line in $\mathbb{P}^2$, and C and F are respectively a section and a fiber of a $\mathbb{P}^1$-bundle on $\mathbb{P}^1 \times \mathbb{P}^1$. In this talk, we will first show that any genus-1 holomorphic Lefschetz pencil is smoothly isomorphic to a blow-up of the either of the two pencils (in $|3H|$ or $|2F + 2C|$).

Relying on braid monodromy techniques due to Moishezon-Teicher, we will further determine vanishing cycles of these Lefschetz pencils.

Jiro Adachi (Hokkaido University)
Goursat surgery and Engel structure

The Goursat flag structure on a manifold is a certain flag of distributions of tangent subspaces. That of length one corresponds to a contact structure on a 3-manifold, and that of length two corresponds to an Engel structure on a 4-manifold. These are the only two, among the Goursat flag structures, that have unique local normal form. Then they are thought of as research objects of global geometry and topology. In this talk, I will introduce a distinctive surgery method of the Goursat manifolds. Especially, I will focus on such surgeries of contact 3-manifolds and Engel 4-manifolds by the Goursat cobordisms, and related things.

Koji Yamazaki (Tokyo Institute of Technology)
Automorphisms of Engel manifolds

An Engel manifold is a 4-manifold with a completely non-integrable 2-distribution called Engel structure. I research the functorial relation between Engel manifolds and Contact 3-orbifolds. And I construct an Engel manifold that the automorphism group is trivial.

Nobuhiro Nakamura (Osaka Medical College)
Homotopy non-equivalence of homeomorphism and diffeomorphism groups of spin 4-manifolds
(joint work with Tsuyoshi Kato and Hokuto Konno)

I will explain that $\mathrm{Diff}(M)$ is not weak homotopy equivalent to $\mathrm{Homeo}(M)$ for $M = K3\#nS^2 \times S^2$ $(0 \leq n \leq 3)$. For the proof, we construct a fiber bundle over T^{n+1} with fiber M whose structure group is $\mathrm{Homeo}(M)$, and show the structure group cannot be reduced to $\mathrm{Diff}(M)$ by using a 10/8-type inequality for spin families obtained from Seiberg-Witten theory. The existence of such a bundle implies that $\pi_i\mathrm{Diff}(M) \neq \pi_i\mathrm{Homeo}(M)$ for some i with $0 \leq i \leq n$.

Nobuo Iida (University of Tokyo)
A Bauer-Furuta type refinement of Kronheimer-Mrowka's invariant for 4-manifolds with contact boundary

The Seiberg-Witten invariant is an invariant for closed 4-manifolds and there are many variants of it. I construct a new variant of the Seiberg-Witten invariant based on two previous works. First, Bauer and Furuta refined the Seiberg-Witten invariant, and made an invariant called the stable cohomotopy invariant, which is an S^1-equivariant stable homotopy map obtained by Furuta's finite dimensional approximation of the Seiberg-Witten map. Second, Kronheimer and Mrowka defined a variant of the Seiberg-Witten invariant for 4-manifolds with contact boundary. I combine these two variants of the Seiberg-Witten invariant; that is, using Furuta's finite dimensional approximation, I refine Kronheimer-Mrowka's invariant for 4-manifolds with contact boundary.

Takayuki Okuda (University of Tokyo)

Topologically inequivalent degenerations with same singular fibers

Namikawa and Ueno completed the classification of degenerations of genus two curves after Kodaira's classification for genus one case. Their paper says that they encountered with the following phenomena, which did not occur in genus one case: there exist some pairs of degenerations that are not topologically equivalent but contain "same" singular fibers. In this talk, we show a necessary and sufficient condition for a degeneration of arbitrary genus curves to be a member of such a pair.

Hokuto Konno (RIKEN)

The diffeomorphism and homeomorphism groups of K3

I shall explain two results on the diffeomorphism and homeomorphism groups of the $K3$ surface. The first result is that the $K3$ surface gives the first counter example to the Nielsen realization problem in dimension 4. The second result is that the natural map $\pi_1(\mathrm{Diff}(K3)) \to \pi_1(\mathrm{Homeo}(K3))$ is not surjective. To our best knowledge, this is the first example of a 4-manifold X for which $\pi_i(\mathrm{Diff}(X)) \to \pi_i(\mathrm{Homeo}(X))$ is not surjective for positive i. The proofs of these results are based on Seiberg-Witten theory. This is joint work with David Baraglia.

Kouichi Yasui (Osaka University)

Stably exotic 4-manifolds

A natural problem in 4-dimensional topology is behavior of smooth structures of 4-manifolds under connected sums. For example, it has been well known that for many families of pairwise exotic compact oriented 4-manifolds, members of each family become pairwise diffeomorphic after taking a connected sum with a fixed 4-manifold such as $S^2 \times S^2$. In this talk, we produce new type of pairwise exotic compact oriented 4-manifolds with respect to their behavior under connected sums. In fact, we give an algorithm that generates large families of such exotic 4-manifolds. We also give their applications.

Benjamin Bode (Osaka University)

A construction of elements of motion groups as algebraic varieties of low degree

The loop braid group is the fundamental group of configurations of an unlink in $\mathbb{R}^3$, where each component lies in a plane parallel to a fixed plane. I will present an algorithm that constructs for every element B of the loop braid group a polynomial map $f : \mathbb{R}^5 \to \mathbb{R}^2$ such that $f^{-1}(0) \cap S^4$ contains the closure of B. Moreover, the algorithm provides an upper bound on the degree of the constructed polynomials. The construction can be generalized to certain elements of other motion groups. We also highlight applications in theoretical physics, in particular a step towards the construction of electromagnetic fields with closed (and potentially knotted) field lines, whose time evolution is described by the chosen element of the motion group.

Seiichi Kamada (Osaka University)

Motion groups of H-trivial links and immersed surface-links

A motion of a link means a one-parameter family of links in 3-space starting from and terminating at the link itself, or an equivalence classes of such families. The set of motions of a given link forms a group, called the motion group. When the link is a trivial link, the motion group was studied by D. Goldsmith. In this talk we discuss motion group for an H-trivial link, which is a disjoint union of some Hopf links and some trivial knots. We give a generating set of the group and an application to immersed surface-links in 4-space. This is a joint work with C. Damiani and R. Piergallini.

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