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Quasi-linear PDEs in fluids

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Abstract. This workshop held by online to conduct international research exchanges on quasi-linear partial differential equations related with fluid mechanics.

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vortex rings, Euler equations, singularity formation, quasi-geostrophic equation, Burgers equation, ill-posedness, martingale solutions

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Preface

This volume of OCAMI Reports is proceedings of the online workshop "Quasi-linear PDEs in fluid" held from February 20th to 21st in 2021. The workshop aimed at exchanging researches on quasi-linear PDEs related to fluid mechanics among young researchers. It was realized by completely online due to COVID-19 though originally planed as a face-to-face meeting at Osaka in the summer of 2020.

The 7 speakers gave 40 mins lectures by zoom. In the 1st day, blow-up and illposedness results on the Euler equations are reported by Profs. Elgindi and Jeong. Profs. Choi and Aiki reported stability and large time behavior of vortex rings. In the 2nd day, Prof. Inahama reported local wellposedness of the stochastic QG equation. Prof. Iwabuchi reported large time behavior of the Burgers equation. Prof. Rongchan Zhu reported non-uniqueness of Markov solutions to the stochastic Euler equations. The program is available at https://sites.google.com/view/qpdes.

The workshop had about 80 participants from Japan, South Korea, India, UK, Canada and US, and conducted international and interdisciplinary exchanges on quasi-linear PDEs.

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Table of Contents

On the Head-on Collision of Coaxial Vortex Rings

Masashi Aiki

The study of the interaction of coaxial vortex rings dates back to the pioneering paper by Helmholtz (1858). Helmholtz considered vortex motion in a incompressible and inviscid fluid based on the Euler equations. His study includes the motion of circular vortex filaments, and he observed that motion patterns such as head-on collision may occur. Since then, many researches have been done on head-on collision of coaxial vortex rings, and interaction of coaxial vortex rings in general. Most of these research are either experiments conducted in a laboratory or numerical simulations of the Navier–Stokes equations, and the rigorous mathematical treatment of head-on collision of vortex rings are very scarce. In light of this, we consider the head-on collision of two coaxial vortex rings, which have circulations of opposite sign, described as the motion of two coaxial circular vortex filaments under the localized induction approximation. A vortex filament is a space curve on which the vorticity of the fluid is concentrated. In our present work, we approximated thin vortex structures, such as vortex rings, by vortex filaments, and described the motion as the motion of a curve in the three-dimensional Euclidean space. In this formulation, a vortex ring is a space curve in the shape of a circle. We prove the existence of solutions to a system of nonlinear partial differential equations modelling the interaction of two vortex filaments proposed by the speaker which exhibit head–on collision. We also give a necessary and sufficient condition for the initial configuration and parameters of the filaments for head-on collision to occur. Our results suggest that there exists a critical value $\gamma_* > 1$ for the ratio γ of the magnitude of the circulations satisfying the following. When $\gamma \in [1, \gamma_*]$, two approaching rings will collide, and when $\gamma \in (\gamma_*, \infty)$, the ring with the larger circulation passes through the other and then separate indefinitely. As far as the speaker knows, the existence of such threshold γ_* is only indirectly suggested via numerical investigations of the head-on collision of coaxial vortex rings, for example by Inoue, Hattori, and Sasaki (2000). Hence, our result is the first to obtain the threshold in a way that is possible to numerically calculate γ_*, as well as prove that the threshold exists in a framework of a mathematical model.

Stability of some exact solutions of incompressible Euler equations

Kyudong Choi

We visit results on stability of exact vortex solutions of the incompressible Euler equations including circular vortex patches, Lamb's dipoles, Hill's spherical vortices. They are solutions minimizing energy under natural constraints so that their stability comes from calculus of variations. This talk is based on joint works with K. Abe (Osaka City Univ.) and with D. Lim (UNIST).

Singularity formation in the 3D Euler equation

Tarek Elgindi

I discuss various issues related to the blow-up problem for the 3D Euler equation.

Paracontrolled quasi-geostrophic equation with space-time white noise

Yuzuru Inahama

We study the stochastic dissipative quasi-geostrophic equation with space-time white noise on the two-dimensional torus. This equation is highly singular and basically ill-posed in its original form. The main objective of the present paper is to formulate and solve this equation locally in time in the framework of paracontrolled calculus when the differential order of the main term, the fractional Laplacian, is larger than 7/4. No renormalization has to be done for this model. (This is a joint work with Yoshihiro Sawano)

Analyticity and large time behavior for the Burgers equation with the critical dissipation

Tsukasa Iwabuchi

We consider the Burgers equation with the critical dissipation.

$$\partial_t u + (-\partial_x^2)^{1/2} + u\partial_x u = 0, \quad t > 0, x \in \mathbb{R}$$
$$u(0,x) = u_0(x)$$

We impose that the initial data is smooth and integrable, and it is known that the solution exists globally in time. In this talk, it will be shown that every solution behaves like the Poisson kernel as time tends to infinity. Analyticity in spacetime will be also discussed.

Ill-posedness for incompressible fluid models at critical Sobolev regularity

In-jee Jeong

We consider the incompressible fluid equations including the Euler and SQG equations in critical Sobolev spaces, which are Sobolev spaces with the same scaling as the Lipschitz norm of the velocity. We show that initial value problem for the equations are ill-posed at critical regularity. This is based on joint works with Tarek Elgindi, Tsuyoshi Yoneda, and Junha Kim.

On ill- and well-posedness of dissipative martingale solutions to stochastic 3D Euler equations

Rongchan Zhu

We are concerned with the question of well-posedness of stochastic three dimensional incompressible Euler equations. In particular, we introduce a novel class of dissipative solutions and show that (i) existence; (ii) weak–strong uniqueness; (iii) non-uniqueness in law; (iv) existence of a strong Markov solution; (v) non-uniqueness of strong Markov solutions; all hold true within this class. Moreover, as a byproduct of (iii) we obtain existence and non-uniqueness of probabilistically strong and analytically weak solutions defined up to a stopping time and satisfying an energy inequality. This talk is based on joint work with Martina Hofmanova and Xiangchan Zhu.