New results on arithmetical complexity of infinite words

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February 17, 2005

Arithmetical complexity $a_u(n)$ of an infinite word $u = u_1 u_2 \cdots u_n \cdots$ is the number of words of length n of the form $u_k u_{k+d} u_{k+2d} \cdots u_{k+(n-1)d}$, where kand d are arbitrary, that is, the number of words of length n which occur in uby arithmetic progressions. It was introduced by Avgustinovich, Fon-Der-Flaass and Frid in 2000 and studied mainly for words whose subword complexity is low, i. e., linear. Arithmetical complexity of such words can behave variously: in particular, it is linear for some Toeplitz words and is exponential for some D0L words.

After a survey of existing results on arithmetical complexity, I would like to give details about recent results on arithmetical complexity of Sturmian words. Although the subword complexity of such words is minimal among non-periodic words and equal to n + 1, their arithmetical complexity has been recently estimated as $a_u(n) = \Theta(n^3)$.

The upper and the lower bounds were obtained by different techniques. The upper bound, obtained by Cassaigne and Frid by the geometric dual method by Berstel and Pocchiola, seems to be closer to reality then the lower one. Additional arguments allowed to extract from this upper bound precise formulas for arithmetical complexity of many Sturmian words, including the Fibonacci word. The lower bound is based on the Fine and Wilf theorem on periodic words.