Let $G_n$ be the group of 2-braid virtual knots of type $(-n, n-1, 1)$, $n \geq 2$, which has 3 virtual crossings, and $G'_n$ its commutator subgroup. Then we show:

$$G'_n \cong \begin{cases} 
\mathbb{Z}_2^n & \text{if } n \equiv 0, 1 \pmod{3}; \\
Q \times \mathbb{Z}_2^{n-2} & \text{if } n \equiv 2 \pmod{3},
\end{cases}$$

where $Q$ is the quaternion group of order 8. In particular, the abelianized group of $G'_n$, $G'_n/G''_n$ is $\mathbb{Z}_2^n$.

According to Shin Satoh, the group of a virtual knot is that of a torus 2-knot in 4-sphere. Also, $G_2$ is the group of the 3-twist spun trefoil knot. However, $G_n$ with $n \geq 3$ is not a 2-knot group. In fact, Hillman has given all possible finite groups that are the commutator subgroups of 2-knot groups, and $G'_n$ with $n \geq 3$ is not contained in his list.

Osaka City University

Date: 5 November 2004.