COMMUTATOR SUBGROUPS OF CERTAIN 2-BRAID VIRTUAL KNOTS

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Let G_n be the group of 2-braid virtual knots of type $(-n, n-1, 1)$, $n \geq 2$, which has 3 virtual crossings, and G'_n its commutator subgroup. Then we show:

$$G'_n \cong \begin{cases} \mathbf{Z}_2^n & \text{if } n \equiv 0,\, 1 \pmod 3; \\ Q \times \mathbf{Z}_2^{n-2} & \text{if } n \equiv 2 \pmod 3, \end{cases}$$

where Q is the quarternion group of order 8. In particular, the abelianized group of G'_n, G'_n/G''_n is $\mathbf{Z}_2^n$.

According to Shin Satoh, the group of a virtual knot is that of a torus 2-knot in 4-sphere. Also, G_2 is the group of the 3-twist spun trefoil knot. However, G_n with $n \geq 3$ is not a 2-knot group. In fact, Hillman has given all possible finite groups that are the commutator subgroups of 2-knot groups, and G'_n with $n \geq 3$ is not contained in his list.

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