Skein relations for the generalized Alexander polynomial for virtual links and closed virtual 2-braids

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Abstract

We show how to find a skein relation for the generalized Alexander polynomial for virtual links.

A quantum invariant for classical links is defined by associating a vector space to a strand. A linear relation among linear maps associated to oriented classical tangles is a skein relation which is helpful for evaluating the invariant. Since a linear map associated to an oriented classical tangle is an intertwiner which is equivariant with respect to the action on $V \otimes V$, we may find a skein relation among any n oriented classical tangles if n is greater than the dimension of the space of intertwiners.

On the other hand, a quantum invariant for virtual links is in the different situation from above. A linear map associated to an oriented virtual tangle is not an intertwiner any more. So we might appreciate an alternative way to find a skein relation for a quantum invariant for virtual links. We focus on the generalized Alexander polynomial for virtual links. We introduce a finite dimensional vector space which includes all linear maps associated to oriented virtual tangles, and gave the dimension of the vector space.

Furthermore, by using this vector space, we find a new skein relation. As an application of the relation, we give a formula for the invariant for a closed virtual 2-braid.