INEQUIVALENT SURFACE-KNOTS WITH THE SAME KNOT QUANDLE

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We have a knot quandle $Q(k)$ and a fundamental class $[k] \in H_2^{\mathrm{Q}}(Q(k); \mathbb{Z})$ as invariants for a classical knot k. Similarly, we have a knot quandle $Q(F)$ and a fundamental class $[F] \in H_3^{\mathrm{Q}}(Q(F); \mathbb{Z})$ as invariants for a surface-knot F.

For classical knots, Joyce and Matveev independently proved that $Q(k)$ characterizes the classical knot k up to reflected inverse, and Eisermann proved that the pair $Q(k)$ and $[k]$ characterize the classical knot k completely.

We consider the following "hierarchy" for surface-knots F and F'.

(i) There exists a quandle isomorphism $\phi : Q(F) \to Q(F')$.

(ii) There exists a quandle isomorphism $\phi : Q(F) \to Q(F')$ such that

$$\phi_*[F] = [F'] \in H_3^{\mathrm{Q}}(Q(F'); \mathbb{Z}).$$

(ii)' There exists a quandle isomorphism $\phi : Q(F) \to Q(F')$ such that

$$\phi_*[F] = \pm[F'] \in H_3^{\mathrm{Q}}(Q(F'); \mathbb{Z}).$$

(iii) The surface-knot F is equivalent to F'.

(iii)' The surface-knot F is equivalent to F' or $-(F')^*$.

We note that (iii) $\Rightarrow$ (ii) $\Rightarrow$ (i) and (iii)' $\Rightarrow$ (ii)' $\Rightarrow$ (i) by definition.

In this talk, we illustrate the gap between (i) and (ii)', the gap between (ii)' and (iii)', and the gap between (ii) and (iii) for surface-knots.

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