In search of an ideal polynomial representation of a knot-type

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Abstract

One of the central themes of *geometric knot theory* is to find an ideal configuration of knot-types in a specified category. Here we consider the space $\mathbb{P}$ of all polynomial knots. Following Vassiliev's notation $\mathbb{P} = K^3 \setminus \Sigma$, where K^3 is the set of all maps $\phi : \mathbb{R} \longrightarrow \mathbb{R}^3$ defined by $\phi(t) = (x(t), y(t), z(t))$ where $x(t) = t^d + a_1 t^{d-1} + \cdots + a_{d-1}t$, $y(t) = t^d + b_1 t^{d-1} + \cdots + b_{d-1}t$, and $z(t) = t^d + c_1 t^{d-1} + \cdots + c_{d-1}t)$ and Σ is the descriminent space of K^3. The topology of the space $\mathbb{P}$ is still under investigation. It is proved that any C^1 knot in S^3 is isotopy equivalent to the closure of the image of such a map for some degree d. Also the path components of $\mathbb{P}$ correspond to a knot-type. It can be easily proved that nontrivial knots cannot be realized by such maps of degree less than 5. As, polynomial represenatation for a given knot-type is not unique, the question of choosing an ideal poynomial representation makes sense. We have made an effort to define an energy function on the space $\mathbb{P}$ and based on this function we call a polynomial representation of a given knot-type with minimum energy to be the ideal one. We shall discuss if such an ideal representation for a given knot-type exists?