ALGEBRAIC EQUATIONS AND KNOT INVARIANTS

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Abstract

In this talk, for a knot K in 3-sphere S^3, we define algebraic varieties $\mathcal{F}^{(d)}(K)$ $(d = 1, 2, 3)$ in a complex space $\mathbb{C}^N$ in the following steps. For a braid presentation σ of a knot K, we first construct finitely many polynomials $\{p_{\sigma,i}\}_i$ on $\mathbb{C}^N$ by using an action of the braid σ on the Kauffman bracket skein module (KBSM) of a handlebody at $t = -1$ with *trace-free condition*. Then the ideal $\mathcal{SL}^{(3)}(\sigma)$ generated by the polynomials $\{p_{\sigma,i}\}_i$ gives an algebraic variety $\mathcal{F}^{(3)}(\sigma)$ via the Hilbert Nullstellensatz. In fact, $\mathcal{F}^{(3)}(\sigma)$ turns out to be invariant under the Markov moves and thus becomes a knot invariant. This is a desired variety $\mathcal{F}^{(3)}(K)$. The above process can be used for *restrictions* $\mathcal{SL}^{(2)}(\sigma)$ and $\mathcal{SL}^{(1)}(\sigma)$ of the ideal $\mathcal{SL}^{(3)}(\sigma)$. Then we can get knot invariants $\mathcal{F}^{(d)}(K)$ $(d = 1, 2)$.

The first variety $\mathcal{F}^{(1)}(K)$ is actually trivial invariant. The third one $\mathcal{F}^{(3)}(K)$ can be considered as a variety containing "a section" of the $SL(2, \mathbb{C})$-character variety of the knot group by using Bullock's theorem (quantization of the $SL(2, \mathbb{C})$-character variety). This view point gives relationships of the variety $\mathcal{F}^{(3)}(K)$ with the number of $SL(2, \mathbb{C})$-irreducible metabelian characters of the knot group (the knot determinant), and moreover the maximal degree (or *span*) of the A-polynomial $A_K(m, l)$ in terms of l, which polynomial is a knot invariant introduced by Cooper, Culler, Gillet, Long and Shalen. Regarding the second variety $\mathcal{F}^{(2)}(K)$, the quotient ring $\mathbb{C}[x_1, \cdots, x_n]/\mathcal{SL}^{(2)}(\sigma)$ $(n \leq N)$ turns out to be isomorphic to the degree 0 knot contact homology which was researched by L. Ng in detail.

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