The growth rate of tunnel numbers of m-small knots

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Abstract

Let $K \subset M$ be a knot in a closed orientable 3-manifold. Let nK be the connected sum of n copies of k, $E(\cdot)$ knot exteriors, and $g(\cdot)$ Heegaard genus. In "On the growth rate of tunnel number of knots" (to appear in *Journal fur die reine und angewandte Mathematik*, available at `http://arxiv.org/abs/math.GT/0402025`) we study the asymptotic behavior of the tunnel number under repeated connected sum operation. We define the growth rate of the tunnel number of K is defined to be:

$$gr_t(k) = \limsup_{n\to\infty} \frac{t(nK) - nt(K)}{n-1}.$$

Let $g = g(E(K)) - g(M)$. Given an integer i $(1 \leq i \leq g)$ let b_i be the bridge index of K with respect to Heegaard surfaces of M of genus $g(E(K))-i$. In this talk we prove that if K is meridionally small (that is, the exterior of K admits no essential meridional surface) than the growth rate of the tunnel number of K is:

$$gr_t(K) = \min_{1\leq i\leq g} 1 - \frac{i}{b_i}.$$

The tools necessary (the strong Hopf–Haken annulus theorem and the Swallow Follow Torus Theorem) will be discussed, as well as some corollaries. Some of the material presented here is still in preparation; some appears in "Heegaard genus of the connected sum of m-small knots" (to appear in *Communication in Analysis and Geometry*, available at `http://arxiv.org/abs/math.GT/0503229`).