Enumerating 3−manifolds by a canonical order II

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This work is a joint work with A. Kawauchi. A well-order (called a *canonical order*) was introduced on the set of links by A. Kawauchi [K]. This well-order naturally induces a well-order on the set of prime link exteriors and eventually induces a well-order on the set of closed connected orientable 3-manifolds.

We assign to every link a lattice point whose length is equal to the minimal crossing number on closed braid forms of the link. We call this number the *length* of the link. We note that a link L is smaller than a link L' in the canonical order if the length of L is smaller than that of L'. We define the *length* of a prime link exterior as the minimal length of a prime link whose exterior is homeomorphic to the given prime link exterior and we define the *length* of a closed connected orientable 3−manifold is the minimal length on prime link exteriors realizing the 3−manifold as the 0 surgery manifold along the prime link.

With respect to the canonical order, we enumerated the prime links with up to length 10 [KT1] and the prime link exteriors with up to length 9 [KT2]. We are now enumerating the 3−manifolds with up to length 9. We classify the manifolds according to their first homology groups. There are 10 types of groups $0, \mathbf{Z}, \mathbf{Z} \oplus \mathbf{Z}, \mathbf{Z} \oplus \mathbf{Z} \oplus \mathbf{Z}, \mathbf{Z} \oplus \mathbf{Z}_2 \oplus \mathbf{Z}_2, \mathbf{Z}_2, \mathbf{Z}_2 \oplus \mathbf{Z}_2, \mathbf{Z}_3 \oplus \mathbf{Z}_3, \mathbf{Z}_4, \mathbf{Z}_4 \oplus \mathbf{Z}_4$ and we have respectively 16,62,16,4,5,7,15,7,5,5 links with these types of groups. We enumerated the manifolds with the group equal to $\mathbf{Z}$ in [KT3]

In this talk, we enumerate the manifolds with the group equal to $\mathbf{Z} \oplus \mathbf{Z}, \mathbf{Z} \oplus \mathbf{Z} \oplus \mathbf{Z}, \mathbf{Z}_2, \mathbf{Z}_2 \oplus \mathbf{Z}_2, \mathbf{Z}_4, \mathbf{Z}_4 \oplus \mathbf{Z}_4$. .

References

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