

A research project

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A spatial graph is a graph embedded in S^3 . If the graph consists of two vertices and three edges such that each edge joins the vertices, then it is called the θ -curve. Also the graph consisting of two loops and an edge joining the vertices of each loop is called the handcuff graph. In my master's thesis, I constructed a notation of a θ -curve by using a tangle and a θ -polyhedron, and enumerated all the prime θ -curves with up to seven crossings. We can enumerate all the prime θ -curves in order of crossing numbers by generalizing Conway's method. Here, a θ -polyhedron is a connected planar graph without bigon, whose two vertices are 3-valent, and the others are 4-valent. Then there exist twenty-four prime basic θ -polyhedra with up to seven 4-valent vertices. We can obtain all the prime θ -curves and handcuff graphs from prime basic θ -polyhedra by substituting tangles for their 4-valent vertices.

I enumerated all the prime handcuff graphs with up to seven crossings in doctoral course first grade. I would like to study the notion of primeness of a handcuff graph. I am planing to make a table of all the prime θ -curves and handcuff graphs with more than seven crossings. In order to do this, I must construct a prime basic θ -polyhedron with more than seven 4-valent vertices, and classify θ -curves and handcuff graphs. I also consider making a table of K_4 graphs embedded in S^3 , because a K_4 graph has four 3-valent vertices. Moreover, I would like to study an invariant which can classify those 3-valent graphs.