Research plan

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1. Generalized spin structures

The fundamental group of $SO(n)$ is isomorphic to $\mathbb{Z}$ for $n = 2$ and $\mathbb{Z}_2$ for $n \geq 3$. Thus $SO(n)$ $(n \geq 2)$ has a non-trivial 2-fold covering. This fact defines the spin structures. Moreover $SO(2)$ has a non-trivial r-fold coverring. Similarly this fact defines the r-spin structures. But it is not possible to define the r-spin structures for $n \geq 3$. I plan to construct the generalized spin structures for the weakly almost complex manifolds, because they have the stable tangent bundle whose structure group is $U(n)$ and the fundamental group of $U(n)$ is isomorphic to $\mathbb{Z}$ for $n \geq 1$. I also plan to construct the cobordism for generalized spin manifolds.

2. Spin mapping class group

Let Σ_g be a compact, oriented surface of genus g. Let Γ_g be the mapping class group of Σ_g. The mapping class group Γ_g acts on an affine space consisting of spin structures on Σ_g. I clarified a relationship between the number of the generators of Γ_g and the spin structures by using this action. If we fix a spin structure σ on Σ_g, then we can consider the diffeomorphisms preserving σ. Let SP_g be the subgroup of Γ_g whose elements leave σ invariant. This group SP_g is called the spin mapping class group. I plan to research the number of the generatos of SP_g by using an action of SP_g.

3. Relationships between the mapping class group and K-theory

M. Atiyah gave the invariant of the spin structure and the spin cobordism by using the mod 2 index and KO group. I plan to clarify relationships between the mapping class group and K-theory by using the spin structures.