Research plan

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Problem

I will study the following problem:

"Determine the universal covering of the Kodaira surface" .

Background of the problem

The above problem is the special cese of Hilbert's 22nd problem "Uniformization problem of complex manifolds."

This 22nd problem is, in the case of complex manifolds of dimension 1 i.e., Riemann surfaces, solved by Koebe and Poincaré. That is, the universal covering of every Riemann surface is biholomorphically equivalent to one of Riemann sphere, complex plane, or upper half-plane.

Next, we consider the case of 2-dimensional complex manifolds. In general, their universal coverings are complicated. For example, Poincaré have showed that the unit ball $B_2 = \{(z_1, z_2) \in \mathbf{C}^2 ||z_1|^2 + |z_2|^2 < 1\}$ is not biholomorphically equivalent to the polydisc $\Delta^2 = \{z_1 \in \mathbf{C}||z_1| < 1\} \times \{z_2 \in \mathbf{C}||z_2| < 1\}$. Thus it is also important to show the universal covering is not biholomorphically equivalent to some simpy connected complex manifold.

So I study the Kodaira surface in the special case of 2-dimensional complex manifold. It is known that the universal covering of the Kodaira surface constructed by Kodaira is not biholomorphically equivalent to B_2. However, determining the universal covering of the Kodaira surface is unsolved.

Research method

I construct the universal covering of the Kodaira surface by using the theory of Teichmüller spaces. However, it is difficult for me to determine the universal covering.

Now, a Kodaira surface of which the fiber is a Riemann surface of type $(3, 0)$ is shown to exist by algebraic method. Then, first I determine the universal covering of the Kodaira surface of type $(3, 0)$, and next work on the determination of type $(g, 0)$.

I can summarize above obsevation as follows.

(1) Understand the existence of the Kodaira surface of type $(3, 0)$ by algebraic method.

(2) Determine its universal covering.

(3) Determine the universal covering of the Kodaira surface of type $(g, 0)$.