Research Result

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My research results are a master's thesis and a solution of a problem raised from there. In this paper, we report the outline of the thesis and the solution of the problem.

Master's thesis

In the master's thesis, we explain the reconstruction of a holomorphic family of Riemann surfaces, which is called a Kodaira surface, by using the theory of Teichmüller spaces.

Here a holomorphic family of Riemann surfaces is defined as $\bigcup_{t\in R}\{t\}\times S_t$ satisfying the following conditions:

(i) R is a Riemann surface.

(ii) S_t is a Riemann surface of type (g,n) which depends holomorphically on any point $t\in R$, where g is the genus of S_t and n is the number of punctures of S_t.

Then $\mathcal{M}=\bigcup_{t\in R}\{t\}\times S_t$ is a 2-dimensional complex manifold, the canonical projection $\pi:\mathcal{M}\to R,(t,p)\to t$ is a holomorphic mapping. We call such a triple $(\mathcal{M},\pi,R)$ a holomorphic family of Riemann surfaces of type (g,n).

A holomorphic family of Riemann surfaces $(\mathcal{M},\pi,R)$ is called a Kodaira suface if it is locally non-trivial, R is compact, and S_t over $t\in R$ is a Riemann surface of type $(g,0)$. Kodaira constructed concretely such a holomorphic family of Riemann surfaces. The important characteristic of the Kodaira surface constructed by Kodaira is that although the base space R and the fiber S_t are compact, singular fibers does not appear as fibres. The Kodaira surface is only such an example, except for a trivial one $(\mathcal{M}_0,\pi_0,R_0)$ such that $\mathcal{M}_0$ is the product of two compact Riemann surfaces R_0,S_0, and $\pi_0:\mathcal{M}_0=R_0\times S_0\to R_0$ is the canonical projection.

Now Kodaira's construction is based on the theory of complex manifolds. On the other hand, following Riera, our construction is done on the theory of Teichmüller spaces as follows.

Let R be a Riemann surface of type $(1,4)$ and S_t be a Riemann surface of type $(2,0)$ which depends holomorphically on any point $t\in R$, and set $\mathcal{M}=\bigcup_{t\in R}\{t\}\times S_t$.

To introduce a 2-dimensional complex structure on $\mathcal{M}$, we use the theory of Teihmüller spaces. The idea is to represent holomorphically $\mathcal{M}$ into Teihmüller space T_2 of genus 2, then to have complex analytic discussions .

A problem and its solution

Kodaira showed that the Kodaira surface is locally non-trivial by using the theory of infinitesimal deformation. Then, is Riera's example locally non-trivial ? I succeeded in proving the problem by using the infinitesimal theory of Teichmüller spaces. The proof was given in the specific case as Riera's example. However, it shows an application of the infinitesimal theory, and is considered to be meaningful.