Research programme

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(1) Kinematic formulae and integral invariants

Based on our previous work we plan to find families of integral invariants for which the kinematic formulae can be expressed in simple terms as in the Chern-Federer formula. Our recent result on kinemaitc formulae in two point homogeneous spaces suggests that the "transfer principle" would hold under some weak conditions. This research leads to the investigation of invariants which have important properties in the sense of geometry. In our previous research we obtained explicit forms of kinematic formulae for integral invariants of degree two. Therefore, by analogy with the proof of some volume minimizing properties of certain submanifolds from the Poincaré formula, it is expected that our new kinematic formulae can be applied to other variational problems, for example the study of Willmore surfaces.

(2) Hamiltonian volume minimizing properties of Lagrangian submanifolds

So far there are only very few non-trivial examples of Lagrangian submanifolds known which minimize volume in their Hamiltonian deformation classes. One reason is the fact that, while Lagrangian intersection theory has been well investigated via Floer homology in symplectic geometry, the tools of integral geometry have not yet been fully exploited. Real forms of Hermitian symmetric spaces are typical examples of minimal Lagrangian submanifolds in Kähler-Einstein manifolds. Hamiltonian stability of real forms of Hermitian symmetric spaces was established by Amarzaya and Ohnita. We propose to investigate their Hamiltonian volume minimizing properties by our new technique. Furthermore, by classifying Hamiltonian volume minimizing Lagrangian submanifolds, we expect to apply our research to the study of moduli spaces of Hamiltonian diffeomorphism classes.

(3) Austere submanifolds and special Lagrangian cones

We determined austere orbits and weakly reflective orbits of s-representations in spheres. We attempt to classify all homogeneous austere submanifolds in spheres. In addition, we attempt to find examples of non-homogeneous weakly reflective submanifolds. It is known that there exist infinitely many non-homogeneous isoparametric hypersurfaces in spheres. We shall invesigate austere submanifolds given as focal submanifolds of these hypersurfaces.

From an austere submanifold in n-dimensional sphere S^n, we can construct a minimal Legendrian submanifold in S^{2n+1} by the twisted normal bundle given by Harvey-Lawson. From this Legendrian submanifold we obtain a special Lagrangian cone in $\mathbb{C}^n$. In recent years, Joyce investigated the smoothing theory of conical singularities on special Lagrangian submanifolds. We plan to study the stabilities and rigidities of special Lagrangian cones which can be obtained from austere orbits of s-representations.

(4) Geometry of submanifolds via integrable systems

Blaschke showed that any parabolic affine sphere in $\mathbb{R}^3$ can be expressed by holomorphic functions. This implies that the Monge-Ampère equation for a parabolic affine sphere is integrable. Recently Cortès-Baues showed that a special Kähler manifold can be realized as a parabolic affine hypersphere. Furthermore, Cortès gave a representation formula for certain parabolic affine hyperspheres. We plan to study parabolic affine hypersurfaces from the viewpoint of integrable systems. We aim to establish a method to constract parabolic affine hyperspheres from holomorphic data as an analogue of the DPW method to construct CMC surfaces. With this method we shall study parabolic affine hyperspheres in the sense of differential geometry. Finally we plan to make new categories of Willmore surfaces, CMC surfaces and affine spheres in the Java version of 3D-XplorMath, a visualization program for mathematical objects.