Research results

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"Integral geometry" is a branch of mathematics which is concerned with the study of certain functions defined on the space of some geometric objects with an invariant measure. The root of integral geometry was in the classical theory of geometric probability, and this developed later into geometric measure theory. Santaló and other geometers applied the integral geometry to isoperimetric inequalities and several geometric variational problems. The purpose of my research is to investigate geometric properties of submanifolds in homogeneous spaces via integral geometry and representation theory of compact Lie groups.

Kinematic formulae are geometric integral formulae averaging invariants of the intersections of submanifolds. A kinematic formula for the volume functional is, in particular, called a Poincaré formula and plays an important role in geometry. Howard obtained a general kinematic formula for integral invariants defined from invariant homogeneous polynomials on the space of second fundamental forms in Riemannian homogeneous spaces. In general it is very hard to find explicit forms of kinematic formulae. In the paper [5], we determined kinematic formulae for integral invariants of degree two in real space forms completely. These are certain extensions of the Chern-Federer formula and Chen's kinematic formula. Furthermore, recently we showed that these kinematic formulae for integral invariants of degree two also hold for hypersurfaces in two point homogeneous spaces. This result is analogous to the "transfer principle" in integral geometry. We will report this in the paper [6].

The kinematic formulae are interesting objects in integral geometry. On the other hand they have remarkable applications to geometric variational problems. Kleiner-Oh showed that totally geodesic $\mathbb{R}P^n$ in $\mathbb{C}P^n$ has Hamiltonian volume minimizing property by using the Poincaré formula for Lagrangian submanifolds in $\mathbb{C}P^n$ and the Arnold-Givental inequality for $\mathbb{R}P^n \subset \mathbb{C}P^n$. Based on their idea, in the paper [3], we introduced a technique using integral geometry and Lagrangian intersection theory to study the problem of Hamiltonian volume minimizing properties of minimal Lagragian submanifolds. In particular, we established the Hamiltonian volume minimizing property of the totally geodesic Lagrangian torus in $S^2 \times S^2$.

The notion of austere submanifolds was introduced by Harvey-Lawson in order to study special Lagrangian cones. From the definition, it is obvious that an austere submanifold is a minimal submanifold. In our recent work, we classified orbits of s-representations, linear isotropy representations of symmetric spaces, which are austere in a sphere. Furthermore, we introduced the notion of weakly reflective submanifold, which is a generalization of the notion of reflective submanifold, and showed that a weakly reflective submanifold is austere. Podestà proved that any singular orbit of a cohomogeneity one action on a Riemannian manifold is an austere submanifold, however, he essentially showed that is a weakly reflective submanifold. We also determined the weakly reflective orbits of s-representations, completely.

International Research Project "Geometry and Visualization"

In the period of 2003–2005 academic year, I was working on the research project "Geometry and Visualization" of Professor Martin Guest in Tokyo Metropolitan University. In this project, we cooperate in the development and the spread of 3D-XplorMath, which is a visualization software designed for the use of research and education in mathematics. In particular, we are in charge of the "Lattice" category in which we shall study non-linear differential equations. For details of this project, please visit our web site:

TMUGS http://tmugs.math.metro-u.ac.jp/