Research plan

Takehisa tsujii

Let $X \in \mathfrak{g}$ be nilpotent, and Λ_X the set of primitive optimal cocharacters for X. For any $\lambda \in \Lambda_X$,

$$m(X, \lambda) = \min\{i \in \mathbb{Z} | X_i \neq \emptyset\}$$

is uniquely determined, say $m(X)$, where $X = \sum_{i \in \mathbb{Z}} X_i$, $X_i \in \mathfrak{g}(i; \lambda)$. Put $k = m(X)$. We can prove that $\Lambda_X = \Lambda_{X_k}$. If $p = \operatorname{char} \Bbbk$ is good for G, this implies that

$$\Lambda'_X = \{\lambda \in \Lambda_X | X \in \mathfrak{g}(k; \lambda)\} \neq \emptyset.$$

Let $\lambda \in \Lambda'_X$. A similar argument of the Premet's proof implies that $G_X = C_G(\operatorname{Im} \lambda)_X \cdot R_u(P(\lambda))_X$ is a Levi decomposition. If $m(X) \leq 2$ for any distinguished nilpotent element X, we can prove the Bala-Carter theorem. We are studying whether we can prove this or not, and whether we can prove the Bala-Carter theorem directly not knowing $m(X) \leq 2$ or not.

Another thing of my interest is to find a briefly proof of the finiteness of numbers of the nilpotent orbits for bad characteristic. The fact of the finiteness is used a proof of the existence of Richardson orbits and regular nilpotent orbits, and so on. In good characteristic, a brief proof of the finiteness was given by Richardson. In bad characteristic, Spaltenstein and others calculated numbers of the nilpotent orbits, but the argument needed higher techniques and was complicated (the Bala-Carter theorem fails in bad characteristic, in fact). If we want to know numbers of the nilpotent orbits explicitly, this argument is reasonable. I wonder that proving only the finiteness of numbers of the nilpotent orbits is so difficult, but is not known an easy proof now.

My research plan is studying ideas of various fields keeping a standpoint on the basis of representations of algebraic groups, and exploit the new field. If I have a chance, I want to tackle the problem of the finiteness of numbers of the nilpotent orbits.