Research Interest and Some Results

Research Interest. My research area lies in complex algebraic geometry and theory of complex manifolds, especially on surfaces of general type . For these few years, I have been studying complex surfaces of general type with small geometric genus, in view of the fundamental groups and the torsion parts of the Picard groups. I am working on, for example, the following problems for minimal regular surfaces X's with each specified first Chern number c_1^2 and Eular characteristic χ of the structure sheaf:

- to clarify what abelian groups appear as the torsion parts of the Picard groups of X's,
- to find, for each abelian group G, all X's whose torsion parts of the Picard groups are isomorphic to G,
- to compute the number of deformation types, or, more generally, to study the moduli space for such X's.

In what follows, we denote by $\mathrm{Tors}(X)$ the torsion part of the Picard group of X, and call it the torsion group.

History. The prototype for such studies, which is due to Y. Miyaoka, M. Reid, and others, is the classification theory for numerical Godeaux surfaces (i.e., minimal surfaces of general type with the first Chern number $c_1^2 = 1$, geometric genus $p_g = 0$, and irregularity $q = 0$). Since the torsion groups for these surfaces coincide with the abelianized fundamental groups, it was effective to employ the torsion group as an additional invariant, in order to distinguish the topological types. Indeed, they first studied possible isomorphism classes of the torsion groups, and then tried to find complete description for the surfaces of each case. One of my motivations for my study is to develop similar stories for surfaces of small geometric genus. Surfaces of general type of small geometric genus, which tend to take several topological types under the same value of the numerical invariants, are interesting objects to study, since they often provide us with exceptions of general fancy theorems.

Some results. So far, I have obtained several results on minimal surfaces X's with $c_1^2 = 2\chi - 1$. Note that this line is parallel to the Noether line, and that the case $\chi = 1$ on this line is that of the Godeaux surfaces.

- In [3], I gave a bound for the orders of the torsion groups for each $\chi \geq 2$,
- In [2], I gave a complete description for the case $\chi = 2$ and $\mathrm{Tors}(X) \simeq \mathbb{Z}/3$: I showed that any such surface is essentially a quotient of a $(3,3)$-complete intersection in $\mathbb{P}^4$, by a certain free action by $\mathbb{Z}/3$, where $\mathbb{P}^4$ denotes the 4-dimensional complex projective space,

- In [4], I showed, for the case $\chi = 2$ and $\mathrm{Tors}(X) \simeq \mathbb{Z}/3$, that any two such surfaces are equivalent under deformation of complex structures (hence diffeomorphic to each other). I also showed that the coarse moduli space for these surfaces is a 14-dimensional unirational variety, and that the infinitesimal Torelli theorem holds at any general points of this moduli space.

- In [3], I constructed examples of the case $\mathrm{Tors}(X) \simeq \mathbb{Z}/2$, for each $2 \leq \chi \leq 4$.

- In [1], I proved that the torsion group of a certain numerical Godeaux surface, which was constructed by E. Stagnaro, is $\mathbb{Z}/5$. Moreover, I showed that the universal cover of this Stagnaro's surface is the quintic surface in $\mathbb{P}^3$ of Fermat type.

The bound for the torsion groups given in [3] imposes a restriction on possible topological types of these surfaces. By using the main theorem of [2], we can obtain any X satisfying the conditions.

Finally, an announcement: I recently succeeded in excluding the case $5 \leq \chi \leq 6$ and $\mathrm{Tors}(X) \simeq \mathbb{Z}/2$, and obtained a complete description for the surfaces of the case $\chi = 4$ and $\mathrm{Tors}(X) \simeq \mathbb{Z}/2$. I am now writing a paper on these new results ([5], in preparation).