Research Plan

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1) The quantum sine-Gordon model is a quantum integrable model which has been studied from various points of view. The S-matrices and form factors were determined by the bootstrap method. These quantities can be understood in terms of the representations of the quantum group $U_q(\widehat{sl}_2)$.

Recently, M. Jimbo and his collaborators studied the space of form factors and its character. They showed that the hidden $U_{q'}(\widehat{sl}_2)$, with a different deformation parameter q', acts not on the form factors but on "deformed cycles" in the integrand of the integral representation of the form factors.

In addition, if we represent the deformation parameter of the first quantum group as $q = e^{i\pi\tau}$, then it is known that the matrix of the Lax operator is related to the universal R-matrix of the third quantum group $U_{\tilde{q}}(\widehat{sl}_2)$ with the modular transformed parameter $\tilde{q} = e^{-i\pi/\tau}$.

Thus, the sine-Gordon model has at least three types of hidden quantum group symmetries. But their relations are not well understood yet. So, I would like to research the relations among these quantum groups from the point view of the "modular double", proposed by L. D. Faddeev.

Faddeev constructed the "modular double" of the quantum group for $U_q(sl_2)$, and determined its universal R-matrix by using quantum dilogarithm functions. By extending to the case of $U_q(\widehat{sl}_2)$, I will try to understand the symmetries of the sine-Gordon model in more unified way.

The study of the quantum dilogarithm functions lead R. M. Kashaev to the "volume conjecture" for three-dimensional hyperbolic manifolds. So, the extension to the case of sine-Gordon may give new insights for the quantum knot invariants.

Moreover, K. Hikami and A. N. Kirillov recently showed that the quantum knot invariant for torus knot has connection with the character of the minimal conformal field theory (CFT). They suggested that hyperbolic manifold may be related to a massive deformation of the CFT.

Since the restricted sine-Gordon model is a massive deformation of a series of the minimal CFTs, it seems important to check whether the character of the deformed cycles is related to the quantum knot invariant for the hyperbolic knots.

2) The Yang-Mills theory has deep connections with many integrable models. The connections have been revealed for various cases, such as for the self-dual Yang-Mills equations, in the high-energy limit, for the Parke-Taylor scattering amplitudes, and for the Seiberg-Witten theory etc.

Recently, a new connection was found for the $\mathcal{N} = 4$ supersymmetric Yang-Mills theory, which is a $3 + 1$ dimensional CFT. The matrix of the anomalous dimension for special types of composite operators is identified with the Hamiltonian of a spin chain model. The Bethe ansatz method was used for the diagonalization of the matrix.

I would like to study to what extent, this new integrable structures are preserved under a (massive) perturbation.