Research plan

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(1) The classification of toric Fano varieties: Smooth toric Fano varieties are classified for dimension at most 4. For the 4-dimensional case, I proved that the smooth toric Fano 4-folds have a "good" birational property which is useful for the classification. However, for the 5-dimensional case, it was shown that we can not expect similar property as the 4-dimensional case. The purpose of this study is to find the "good" law for the smooth toric Fano 5-folds and higer-dimensional cases, and apply the classification problem.

(2) The embedding problem of abelian surfaces: We can determine this problem for about 100 smooth toric Fano 4-folds. For the left cases, we need more complicated techniques. We are studying this problem by using PC program such as Macaulay. It is convinient to calculate the Chow ring of a toric variety.

(3) The classification of 4-dimensional toric morphisms whose canonical divisor is anti-nef: For this problem, we have at least two meaning. The first one is to apply the result to the classification of smooth toric weak fano 4-folds. This means that by patching "local" weak Fano 4-folds we can obtain a smooth toric weak Fano 4-folds. The second one is to apply the 3-dimensional hypersurface canonical singulariteis. Since the ADE singulariteis are very important in the algebraic geometry, it is very important to study the 3-dimensional hypersurface canonical singulariteis which are higer-dimensional version of the ADE singularities.

(4) The classification of toric Fano manifolds and toric manifolds by "coindex": Toric Fano manifolds of coindex at most 5 are completely classified. So, the next step is the classification of toric Fano manifolds of coindex 6. In this case, the types of extremal rays are well-known. Therefore, I think that by using the toric Mori theory deeply, we can complete the classification soon. On the other hand, compact toric manifolds of coindex at most 2 are completely classified. So, naturally, we think that we want to classify compact toric manifolds of coindex 3. Since there exist infintely many such manifolds, the classification is much more complicated than the case of Fano manifolds. However, it is rather easy to classify "projective" toric manifolds of coindex 1 or 2. Therefore, it is expected that the classification is likely to be realized under the assumption that manifolds are projective, that is, we can apply the toric Mori theory.