Summary of my research

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Knot theory is the theory of the mathematical study of knots to investigate knotted simple closed curves in a 3-dimensional space. Though there are a lot of ways to study, I regard knot theory as a field of geometric topology. I carried out researches concerning knot cobordism for 10 years. Knot cobordism is a field of knot theory to study knots as boundary of a surface in a 4-dimensional space. Now I explain my old research.

1. 1996-1999

I studied knot cobordism in terms of the properties of quasipositive knots. Particularly, I showed the existence of an infinite family of linearly independent knots with arbitrary gaps between 4-genera and topological 4-genera by showing important properties of quasipositive knots concerning knot cobordism. Maximal Thurston-Bennequin number is an invariant of knots and links derived from 3-dimensional contact geometry. I gave certain formula concerning Kauffman polynomials and Maximal Thurston-Bennequin numbers of positive knots and links. Moreover, I generalized the result to alternating knots and links.

2. 1999-2002

By studing the properties of the Jones polynomials of symmetric unions of knots, I gave an infinite family of counterexamples for a conjecture of T. Fiedler, given in 1999, about the cobordism invariance of the Jones polynomial of a knot. Furthermore, by developing the research, I gave a classification of 2-knots (embedded spheres in a 4-dimensional space) associated with symmetric unions.

3. 2002-2003

I researched on a linear representation of the fundamental group of the knot complement of a knot. The A-polynomial of a knot is an invariant derived from all $SL(2, C)$-representations of the fundamental group of the knot complement. The problem that whether the knot which gives the trivial A-polynomial is the unknot was given in a paper of D. Cooper and D. Long. This problem had already been proved affirmatively for knots except satellite knots with trivial Alexander polynomials. I showed the existence of an infinite family of prime satellite knots with the trivial Alexander polynomials and the nontrivial A-polynomials.

4. 2003-2004

I carried out a research about a colored Jones polynomial of a knot. Now it is considered to be important to find a formula for the colored Jones polynomial because it contributes to investigating the Kashaev-Murakami-Murakami volume conjecture. I have obtained a formula for the N-colored Jones polynomial of a double of a knot K in terms of the colored Jones polynomial of K by using skein theory. It generalizes Masbaum's formula for K being the unknot. As a corollary, we show that if the volume conjecture for untwisted doubles of knots is true, then the colored Jones polynomial of a knot detects the unknot.

5. 2004-2005

I investigated a relationship between knot cobordism and a clasper. A clasper is an important object in a study of finite type invariants. I showed that if a knot K is obtained from K' by using a graph clasper whose all components have the positive first Betti numbers, then there is a ribbon concordance from K' to K.

6. 2005-2006

I confirmed a question of E. Ferrand concerning a relation between HOMFLY polynomials and

Kauffman polynomials. By collaborating with A. Stoimenow and D. Matei, we gave counterexamples for a problem of Przytycki (Kirby's problem 1.91(2)) concerning the colored Jones polynomials and mutations of a knot. On the one hand, I have shown the existence of an exotic structure of a certain Casson handle topologically using Rasmussen invariant.