

The plan of our study

Every link can be realized as a closed braid. A. Kawauchi suggested the following project: Enumerate the prime links by using the braid expressions and enumerate the 3-manifolds by using the enumeration of the links. Our main purpose is to complete the table of 3-manifolds according to this project.

We describe the outline of the project. A well-order (called a *canonical order*) was introduced in the set of links by A. Kawauchi [K] (see also A. Kawauchi and I. Tayama [KT]).

We assign to every link a lattice point whose length is equal to the minimal crossing number on closed braid forms of the link and we call the number the *length* of the link. We note that a link L is smaller than a link L' in the canonical order if the length of L is smaller than that of L' , and for any natural number n there are only finitely many links with lengths up to n . Let \mathbf{L}^p be the set of prime links and \mathbf{M} the set of closed connected orientable 3-manifolds. Let $\chi : \mathbf{L}^p \rightarrow \mathbf{M}$ be a map defined by $\chi(L) = \chi(L, 0)$ (this means the result of the 0-surgery on S^3 along L). Then it is known that χ is surjective and A. Kawauchi defined a map $\alpha : \mathbf{M} \rightarrow \mathbf{L}^p$ by $\alpha(M) = \min\{L \in \chi^{-1}(M) : L' \in \chi^{-1}(M), \pi_1(E(L)) = \pi_1(E(L')) \Rightarrow L < L'\}$ for $M \in \mathbf{M}$, where $E(L)$ is the exterior of L .

By using α , we consider \mathbf{M} as a subset of \mathbf{L}^p and introduce the well-order into \mathbf{M} . We have finished enumerating the links with lengths up to 10. By using this result, we will enumerate the closed connected orientable 3-manifolds with lengths up to 10, that is, arrange the points in $\alpha(\mathbf{M})$ under this order. You might think it is easy. But actually it is quite time-consuming, because we don't have an algorithm to determine the value of $\alpha(M)$ for $M \in \mathbf{M}$ even though it is well defined. A. Kawauchi has proposed a plan of determining $\alpha(M)$.

References

[K] A. Kawauchi, A tabulation of 3-manifolds via Dehn surgery, Boletín de la Sociedad Matemática Mexicana (to appear).

[KI] A. Kawauchi and I. Tayama, Enumerating prime links by a canonical order, J. Knot Theory and Its Ramifications (to appear).