Abstract for our study

1. Our study on line configurations in complex planes

Let $l = l_1 \cup l_2 \cup \cdots \cup l_\mu$ be a collection of straight lines $\mathbf{R}$'s in $\mathbf{R}^2$. Each line is of the form $l_i = \{(x, y) \in \mathbf{R}^2 | a_i x + b_i y + c_i = 0\}$ for real numbers a_i, b_i and c_i. Let $L_i = \{(x, y) \in \mathbf{C}^2 | a_i x + b_i y + c_i = 0\}$ and we have $L = L_1 \cup L_2 \cup \cdots \cup L_\mu$, a collection of $\mathbf{C}$'s in $\mathbf{C}^2$. This is called a real line configuration in $\mathbf{C}^2$. The complex projective plane $\mathbf{CP}^2$ is the quotient space of $\mathbf{C}^3$ by identifying (x, y, z) and $\lambda(x, y, z)$ for complex numbers x, y, z and λ. Let $\mathcal{L}_i = \{[x, y, z] \in \mathbf{CP}^2 | a_i x + b_i y + c_i z = 0\}$, then we have $\mathcal{L} = \mathcal{L}_1 \cup \mathcal{L}_2 \cup \cdots \cup \mathcal{L}_\mu$, a collection of $\mathbf{CP}^1$'s in $\mathbf{CP}^2$. This is called a real line configuration in $\mathbf{CP}^2$.

First we describe our study on a real line configuration $\mathcal{L}$ in $\mathbf{CP}^2$. We proved the following results concerning the first Betti numbers of abelian coverings of $\mathbf{CP}^2$ branched over real line configurations:

(1) An estimate of the first Betti numbers .

(2) A characterization of a central and general position line configurations in the terms of the first Betti numbers of abelian coverings.

(3) The first Betti numbers of the abelian coverings of the real line configurations up to 7components.

Next we describe our study on a real line configuration in $\mathbf{C}^2$. For a real line configuration L, we construct a ribbon surface-link which has the same group as L. If L is a central or general position line configuration, the constructed ribbon surface-link has the minimum genus.

2. Our study on links in the three dimensional sphere

First we describe our study on 2 component links. We give a formula to express the first homology groups of the $\mathbf{Z}_2 \oplus \mathbf{Z}_2$ branced coverings of $L = K_1 \cup K_2$ in terms of those of three smaller cyclic branched coverings.

Next we describe our study on a table of links. A. Kawauchi defined a well-order in the set of links which induces a well-order in the set of 3-manifolds. In fact, he enumerated the first 28 prime links and the first 26 manifolds. We extended the table of links and enumerated the first 443 prime links. Our argument enables us to discover 7 omissions and 1 overlap in Conway's table of prime links of 10 crossings.